Here are some facts about the hyperbolic plane.

**Theorem 1.** Given two rays with a common vertex, there is a unique line parallel to both rays.

**Theorem 2.** Every omega triangle has finite area. Two “congruent” omega triangles have the same area.

**Theorem 3.** Suppose $f$ is a continuous real valued function defined on an interval containing arbitrarily small numbers (e.g. $(0, \pi)$) and that $f$ satisfies $f(x + y) = f(x) + f(y)$ whenever both sides are well defined. Then there is a constant $c$ such that $f(x) = cx$.

1. Show that two 2/3 ideal triangles with the same interior (or exterior) angles have the same area.

2. Let $A(\alpha)$ denote the area of a 2/3 ideal triangle with exterior angle $\alpha$. Use the diagram below to conclude that $A(\alpha + \beta) = A(\alpha) + A(\beta)$.

3. Conclude that there is a constant $I$ such that $A(\alpha) = I\alpha$.

4. Show that any two ideal triangles have the same area. Relate the area of an ideal triangle to the constant $I$ from problem 3.

5. Consider the diagram below. Find a formula that relates the area of the triangle to the sum of the interior angles of the triangle.