Quiz 6 Name: Solutions

Math 202 F01 03/10/06

1. Let $f(x,y) = x^3y + 12x^2 - 8y$. Determine all critical points of f, and (if possible) determine whether they are local maxima, minima, or saddle points.

$$\nabla f = (3x^2y + 24x,\ x^3 - 8) = (0,0)$$

$$x^3 - 8 = 0 \Rightarrow x = 2$$

$$3x^2y + 24x = 0 \Rightarrow 12y + 48 = 0 \Rightarrow y = -4$$

$(2,-4)$ is the only critical pt.

2^{nd} Deriv. Test: $f_{xx}\big|_{(2,-4)} = 6xy + 24\big|_{(2,-4)} = -24$

$$D\big|_{(2,-4)} = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 6xy+24 & 3x^2 \\ 3x^2 & 0 \end{vmatrix} = -9x^4\big|_{(2,-4)} = -144 < 0$$ so saddle

2. Suppose you want to find the minimum of $f(x,y,z) = x^2 - 2y^3z$, but under the constraint that x, y, z satisfy $x^2 + y^2 = xyz$.

   (a) What system of algebraic equations should be solved to find all candidates for the minimum? (Do *not* attempt to solve the system.) $g(x,y,z) = x^2 + y^2 - xyz = 0$ is constraint

   $$\nabla f = \lambda \nabla g \Rightarrow (2x, -6y^2z, -2y^3) = \lambda(2x - yz,\ 2y - xz,\ -xy)$$

   $$2x = \lambda(2x - yz)$$
   $$-6y^2z = \lambda(2y - xz)$$
   $$-2y^3 = \lambda(-xy)$$
   $$x^2 + y^2 = xyz$$

   (b) If you had a list of all solutions to the equations in part (a), what would you have to do to find the minimum of f?

   Just evaluate f at each of the points, + pick the one(s) giving the smallest value.