Quiz 5 Name: Solutions

Math 202 F01 03/03/05

1. For the function $f(u,v) = uv - e^{3u}v + \cos(\pi u)$, give a linear approximation valid near $(u,v) = (0,2)$.

$$\left.\frac{\partial f}{\partial u}\right|_{(0,2)} = v - 3e^{3u}v - \pi\sin(\pi u)\Big|_{(0,2)} = 2-6-0=-4$$

$$\left.\frac{\partial f}{\partial v}\right|_{(0,2)} = u - e^{3u}\Big|_{(0,2)} = 0-1=-1$$

$$f(0,2) = 0-2+1=-1$$

so $L(x,y) = -1 + -4(u-0) + -1(v-2)$

2. For the function $T(x,y,z) = \dfrac{zy}{1+x^2}$,

(a) Find the differential dT when $(x,y,z) = (1,3,2)$

$$\left.\frac{\partial T}{\partial x}\right|_{(1,3,2)} = \frac{-zy}{(1+x^2)^2}(2x)\Big|_{(1,3,2)} = \frac{-6}{2^2}(2) = -3$$

$$\left.\frac{\partial T}{\partial y}\right|_{(1,3,2)} = \frac{z}{1+x^2}\Big|_{(1,3,2)} = \frac{2}{2} = 1$$

$$\left.\frac{\partial T}{\partial z}\right|_{(1,3,2)} = \frac{y}{1+x^2}\Big|_{(1,3,2)} = \frac{3}{2}$$

so $dT = -3dx + 1dy + \frac{3}{2}dz$

(b) Near $(1,3,2)$, is the function more sensitive to small changes in x, y, or z ? Briefly explain your reasoning.

Most sensitive to small changes in $\underline{x}$, since the coefficient for dx is larger (in absolute value) than that of dy or dz in the differential dT.