

1. For each of the following limits, either determine its value, or show that it does not exist.

(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + 2y^2}{2x^2 + y^2}$

As $(x,y) \rightarrow (0,0)$ along the x -axis, so $y=0$, $\frac{x^2 + 2y^2}{2x^2 + y^2} = \frac{x^2}{2x^2} = \frac{1}{2}$
so the function $\rightarrow \frac{1}{2}$

As $(x,y) \rightarrow (0,0)$ along the y -axis, so $x=0$, $\frac{x^2 + 2y^2}{2x^2 + y^2} = \frac{2y^2}{y^2} = 2$
so the function $\rightarrow 2$

Since $\frac{1}{2} \neq 2$, this limit **D.N.E.**

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + xy^2}{x^2 + y^2}$

Since $\frac{x^3 + xy^2}{x^2 + y^2} = \frac{x(x^2 + y^2)}{(x^2 + y^2)} = x \rightarrow 0$ as $(x,y) \rightarrow (0,0)$

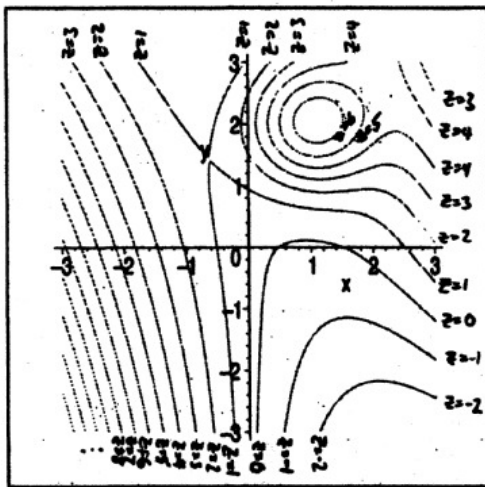
The limit is \circ

(c) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}$

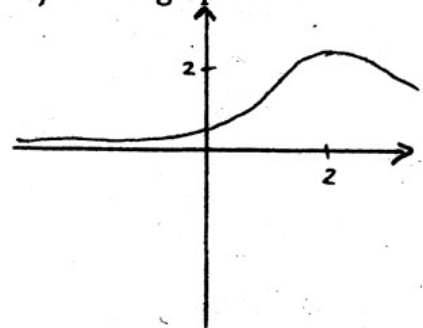
As $(x,y) \rightarrow (0,0)$ along x -axis $\frac{x^2}{x^2 + y^2} = \frac{x^2}{x^2} = 1 \rightarrow 1$
As $(x,y) \rightarrow (0,0)$ along y -axis $\frac{x^2}{x^2 + y^2} = \frac{0}{y^2} = 0 \rightarrow 0$

Since $1 \neq 0$, this limit **D.N.E.**

2. A contour plot for a function $z = f(x,y)$ is shown. On the axes provided, draw plots of the cross-sections (or traces) of the graph where $x = 0$ and where $y = 0$.



$x=0$:



$y=0$:

