

Quiz 1
Math 202 F01

Name: Solutions
01/26/06

A 25 lb. object is located at the point $A = (1, 0, 4)$, but is constrained so that it can only move along a straightline track toward the point $B = (0, -2, 2)$.

1. Give, in coordinate form, a *unit* vector representing the direction in which the object can move.

$$\vec{v} = B - A = (0, -2, 2) - (1, 0, 4) = (-1, -2, -2)$$

$$\|\vec{v}\| = \sqrt{1+4+4} = 3$$

$$\text{So } \vec{u} = \frac{1}{3}\vec{v} = \left(-\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}\right)$$

2. Give, in coordinate form, a vector representing the force of gravity on the object.

$$\vec{F} = (0, 0, -25) \text{ lbs}$$

3. What is the angle between the direction the object can move and the gravitational force vector? (Give an exact answer, possibly involving inverse trigonometric functions).

$$\vec{u} \cdot \vec{F} = \|\vec{u}\| \|\vec{F}\| \cos \theta$$

$$\frac{50}{3} = (1)(25) \cos \theta$$

$$\frac{2}{3} = \cos \theta$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

4. Find a vector expressing that part of the gravitational force that could cause the object to move. Specify units.

$$\text{proj}_{\vec{v}} \vec{F} = \frac{\vec{v} \cdot \vec{F}}{\vec{v} \cdot \vec{v}} \vec{v} = \frac{50}{9} (-1, -2, -2) = \left(-\frac{50}{9}, -\frac{100}{9}, -\frac{100}{9}\right) \text{ lbs}$$

or

$$\text{proj}_{\vec{u}} \vec{F} = \frac{\vec{u} \cdot \vec{F}}{\vec{u} \cdot \vec{u}} \vec{u} = \frac{50}{1} \left(-\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}\right) = \left(-\frac{50}{9}, -\frac{100}{9}, -\frac{100}{9}\right) \text{ lbs}$$