SOME SELECTED HW SOLUTIONS HW 1 MATL 660

1. a. T_2, T_3 b. T_2, T_3, T_5 c. T_2, T_3, T_5 and T_1, T_6 d. all e. T_4, T_6

4. ... $b(10) = 2{,}027{,}025$

5. $b(n) = (2n-5)!! = (2n-5)(2n-7)(2n-9)(2n-11)\cdots 5(3)(1)$

$$= \frac{(2n-5)(2n-6)(2n-7)(2n-8)\cdots(4)(3)(2)(1)}{(2n-6)\quad(2n-8)\quad\cdots\quad(4)\quad(2)}$$

$\swarrow (2n-5)!$ ← product of evens

$$= \frac{(2n-5)!}{2(n-3)\,2(n-4)\cdots 2(2)\,2(1)}$$

$\hookrightarrow (n-3)!$ (product of $n-3$ 2s)

$$= \frac{(2n-5)!}{2^{n-3}(n-3)!}$$

7. Cann et al produced rooted trees for $n = 147 \Rightarrow (2n-3)!! = 291!!$ trees

If you computed the number of unrooted trees, then $(2n-5)!! = 289!!$

$$(2n-3)!! = \frac{(2n-3)!}{2^{n-2}(n-2)!} = \frac{291!}{2^{145}\,145!} \approx 4.89 \times 10^{296}$$

From prob 5, $289!! = \dfrac{289!}{2^{144}(144!)} \approx 1.68 \times 10^{294}$

In case your calculator balked, a nifty trick is to use logs:

For instance, $$\frac{291!}{2^{145}\,145!} \sim \frac{\cancel{\sqrt{2\pi}}\,291^{291.5}\,e^{-291}}{2^{145}\,\cancel{\sqrt{2\pi}}\,145^{145.5}\,e^{-145}}$$ by Stirling's Formula

Applying a log $\Rightarrow$ $291.5\ln(291) - 291 - 145\ln 2 - 145.5\ln 145 - 145 \approx 683.1526$

Taking the exponential $\Rightarrow$ the number of trees $\approx \exp(683.1526)$
$= 4.8912 \times 10^{296}$