

Mathematics 421: Applied Analysis  
Fall 2005

**Instructor:** John Rhodes

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**Office Hours:** M 11:45-12:45, W 2:15-3:15, F 10:30-11:30, and by appointment

**Web page:** <http://www.dms.uaf.edu/~jrhodes/M421.html>

**Prerequisites:** Math 302, and indirectly Math 202. You may find you need to review Math 202 material very early in this course.

**Credit Hours:** 4.0

**Texts:** Div, Grad, Curl, and all that, 4th ed., by H. M. Schey, Norton;  
Fourier Series and Boundary Value Problems, 6th ed., by J. W. Brown  
and R. V. Churchill, McGraw Hill

**Class Meetings:** M W F 9:05-10:15 in 410 Gruening, T 9:45-10:45, in 106  
Chapman

**In-class Hour Exams:** Friday, October 7; Friday, November 11

**Final Exam:** 8:00-10:00, Friday, December 16

**Course overview and goals:**

‘Analysis’ is the sophisticated word used to describe mathematics based in the ideas arising in calculus. In ‘Applied Analysis’ our goal is not to develop the theory of analysis for its own sake, but rather to see how analysis can provide us with both understanding and tools for the real world. While there will be ‘theory’ in the course, it is always theory that is needed for what are ultimately quite concrete problems.

The course consists of two rather distinct parts, though they are interrelated.

In the first third of the course we will use the Schey text as a guide to exploring vector analysis in three dimensional space. You have been exposed to the basics of this material in Multivariable Calculus (Math 202), and perhaps a bit more in physics or engineering courses. We’ll develop the key theorems relating differentiation and integration in many dimensions, aiming for both strong conceptual understanding and calculational ability. This material is vital for mathematical treatments of both physical forces (electrical, gravitational, etc.) and fluid flows (including air flow), and also has an appealing mathematical structure. As such, its good background for any mathematician, physical scientist, or engineer.

The remainder of the course is an introduction to partial differential equations (PDEs). While a few of these equations (the heat equation, the wave equation) are quite famous, in fact myriad PDEs arise throughout the sciences, both pure and applied. Many applied mathematicians, and many mathematically-inclined scientists and engineers, spend much of their careers solving, or trying to solve, PDEs. We'll explore some of the more famous PDEs, and discuss various approaches to solutions. In particular, we'll develop the theory of Fourier analysis as one means of solution. Although applicable only in limited settings, Fourier analysis is an extremely powerful conceptual framework and practical tool.

If you complete this course successfully, you will have a good command of the basic mathematical framework of vector analysis and PDEs. You should be able to apply appropriate mathematical tools to understand and solve standard problems in these areas drawn from the physical sciences or engineering. You will have an appreciation for the conceptual power of the mathematical theory, as well as some understanding of the need for rigorous proof, even though you will not have focused on developing proofs yourself.

**Mechanics of the course:**

The class will be run as an interactive lecture. That means that while I will be presenting material at the board, and you will be taking notes, I will also be asking for suggestions, ideas, and questions about the material as we go along. I don't expect 'correct' answers, but I do expect you to be actively following and participating — that makes the class more interesting for us all.

Homework will usually be assigned daily, but only collected each Monday. I will typically begin each class by asking if there are questions about the last lecture and its homework assignment. That means you should review notes and make an initial attempt on homework problems before the next class meeting, even though problems may not be collected until several days later. While it never hurts to ask, in general I will defer questions about any earlier assignment to my office hours, in order to keep the course moving along.

Although I will not formally take attendance, obviously it will factor into your evaluation in class participation. If you miss class, you should get notes from another student, and can find homework assignments posted on the course web page.

I will not accept *any* late homework that has not been cleared ahead of time or is not due to a genuine emergency (e.g., a death in the family). Even though you may find you can't do every problem, you must make a reasonable attempt on them all.

I encourage you to work with others on the homework, *but to write up the solutions independently*. In writing up your work, you should present your arguments in such a way that an intelligent, but ignorant, person can understand them. In particular complete sentences and a logical presentation are expected.

The entire homework assignment will be checked to be sure you have attempted everything. Selected problems may be graded more completely.

For missed examinations that are not approved in advance, no make-up exams will be given except in extreme circumstances (e.g., family death, documented illness, etc.)

**Grades:**

Your performance will be evaluated based on 10% homework, 5% class participation, 25% first midterm exam , 25% second midterm exam, 35% final exam.

Course grades will be determined according to the following cutoffs:

$$A \geq 90\%,$$

$$B \geq 80\%,$$

$$C \geq 70\%,$$

$$D \geq 60\%.$$

I reserve the right to move the cutoff points downward if particular exams turn out to be unexpectedly difficult. Note that you are not in competition with your peers – everyone in the class may get an *A*, or everyone may get an *F*.

**Univerisity and Department Policies:**

Your work in this course is governed by the UAF Honor Code. The Department of Mathematics and Statistics has specific policies on incompletes, late withdrawals, and early final exams which can be found at

<http://www.dms.uaf.edu/dms/Policies.html>.

If you have any disabilities that I should know about, you should bring them to my attention soon so that we can work with the Office of Disability Services to set up any necessary accommodations.

## Tentative Schedule

Week 1	Sept. 2 – Sept. 9	Vector Functions, <i>Labor Day</i> , Surface Integrals
Week 2	Sept. 12 – Sept. 16	Divergence, Curvilinear coordinates, the Divergence Theorem, Line integrals
Week 3	Sept. 19 – Sept. 23	Line integrals and path independence, Curl, Stokes' Theorem
Week 4	Sept. 26 – Sept. 30	Gradients, Laplace's Equation
Week 5	Oct. 3 – Oct. 7	Partial Differential Equations (PDEs) and Boundary Value Problems (BVPs), the Heat and Wave equations, EXAM 1
Week 6	Oct. 10 – Oct. 14	Linearity and superposition, introduction to Fourier's method
Week 7	Oct. 17 – Oct. 21	Fourier series – basic theory
Week 8	Oct. 24 – Oct. 28	Fourier series – convergence issues
Week 9	Oct. 31 – Nov. 4	Applications of Fourier series to BVPs
Week 10	Nov. 7 – Nov. 11	Sturm-Liouville theory, EXAM 2
Week 11	Nov. 14 – Nov. 18	Sturm-Liouville applications
Week 12	Nov. 21 – Nov. 23	Fourier Transforms, <i>Thanksgiving</i>
Week 13	Nov. 28 – Dec. 2	Fourier Transforms (cont.)
Week 14	Dec. 5 – Dec. 12	extra time for catching-up or additional topics