Quiz 6
Math 314

Name: Key

3/31/08

1. The matrix

$$A = \begin{pmatrix} -1 & -2 & -2 & -5 & -6 \\ 1 & 2 & 5 & 14 & 2 \\ -2 & -4 & -1 & -1 & -16 \\ 1 & 2 & 1 & 2 & 3 \end{pmatrix}$$

has reduced row echelon form

$$U = \begin{pmatrix} 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

(a) Give a basis for the column space of A.

$\begin{pmatrix} -1 \\ 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 5 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -6 \\ 2 \\ -16 \\ 3 \end{pmatrix}$, the pivot columns of $\underline{A}$

(b) Give a basis for the row space of A.

$\begin{pmatrix} 1 \\ 2 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$, the pivot rows of $\underline{U}$

(c) What is the dimension of the nullspace of A?

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2. Suppose a 57×32 matrix B has rank 32.

(a) Will the system of equations $B\mathbf{x} = \mathbf{b}$ be solvable for all $\mathbf{b} \in \mathbb{R}^{57}$? Why or why not?

No $\quad Col(B)$ is a 32-dim. subspace of $\mathbb{R}^{57}$ and $B\vec{x} = \vec{b}$ is solvable only for $\vec{b} \in Col(B)$ —OR— G.E on B produces rows of zeros

(b) Suppose for a particular $\mathbf{b}$ the system $B\mathbf{x} = \mathbf{b}$ has a solution. Will the solution be unique? Why or why not?

Yes $\quad Nul(B)$ is 0-dimensional, so $Nul(B) = \{\vec{0}\}$ and any 2 solutions must differ by an element of $Nul(B)$

—OR— G.E. on B produces no free variables