Quiz 4 Name: Key

Math 314 2/25/08

1. Let $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & -1 \\ -1 & 3 & 1 \end{pmatrix}$. Find A^{-1}, or indicate that it doesn't exist.

   Show all your work.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 2 & 4 & -1 & 0 & 1 & 0 \\ -1 & 3 & 1 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -2 & 1 & 0 \\ 0 & 5 & 1 & 1 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 5 & 1 & 1 & 0 & 1 \\ 0 & 0 & -1 & -2 & 1 & 0 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 5 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 5 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/5 & 1/5 & 1/5 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 7/5 & -2/5 & -2/5 \\ 0 & 1 & 0 & -1/5 & 1/5 & 1/5 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right) \quad \text{so } A^{-1} = \begin{pmatrix} 7/5 & -2/5 & -2/5 \\ -1/5 & 1/5 & 1/5 \\ 2 & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1.4 & -.4 & -.4 \\ -.2 & .2 & .2 \\ 2 & -1 & 0 \end{pmatrix}$$

2. If $T : \mathbb{R}^a \to \mathbb{R}^b$ is a linear transformation, answer the following:

   (a) The matrix A for which $T(\mathbf{x}) = A\mathbf{x}$ will be of what size?

   $b \times a$

   (b) If T is onto, what can you say about the number of pivots of A?

   # of pivots $= b$, since $A\vec{x}=\vec{b}$ is soluble for every $\vec{b}$ means there is a pivot in every row

   (c) It T is one-to-one, what can you say about the number of pivots of A?

   # of pivots $= a$, since $A\vec{x}=\vec{b}$ has at most one solution means there is a pivot in every column.