Quiz 3 Name: Key

Math 314 2/15/08

1. Let $\mathbf{a} = (-1,\ 2,\ 1)$, $\mathbf{b} = (2,\ 1,\ -3)$, and $\mathbf{c} = (4,\ 7,\ -7)$.

   (a) Are these three vectors linearly independent or dependent? Show enough work to justify your answer.

   Solve $x\vec{a} + y\vec{b} + z\vec{c} = \vec{0}$:

   $$\begin{pmatrix} -1 & 2 & 4 \\ 2 & 1 & 7 \\ 1 & -3 & -7 \end{pmatrix} \rightarrow \begin{pmatrix} -1 & 2 & 4 \\ 0 & 5 & 15 \\ 0 & -1 & -3 \end{pmatrix} \rightarrow \begin{pmatrix} -1 & 2 & 4 \\ 0 & 5 & 15 \\ 0 & 0 & 0 \end{pmatrix}$$

   There is a free variable, hence non-trivial solutions exist.

   Thus the vectors are <u>dependent</u>.

   (b) What is the geometric meaning of your answer to part (a)? I.e., what does it tell you about how these vectors are located in $\mathbb{R}^3$?

   One of the vectors lies in the plane spanned by the other two.

   Equivalently, the three vectors all lie in a plane.

2. A linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ is such that for $\mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$,

   $T(\mathbf{v}) = \begin{pmatrix} -5 \\ 7 \end{pmatrix}$ and for $\mathbf{w} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$, $T(\mathbf{w}) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

   If $\mathbf{x} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = 2\mathbf{v} - \mathbf{w}$, what is $T(\mathbf{x})$?

   $T(\vec{x}) = T(2\vec{v} - \vec{w}) = 2T(\vec{v}) - T(\vec{w})$ by linearity

   $$= 2\begin{pmatrix} -5 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \underline{\begin{pmatrix} -12 \\ 13 \end{pmatrix}}$$