Quiz 2
Math 314

Name: Key

2/6/08

Suppose a system of equations expressed by $A\mathbf{x} = \mathbf{b}$ is given. A and $\mathbf{b}$ remain the same for all questions below.

1. When Gaussian elimination is performed on the augmented matrix $(A|\mathbf{b})$, it leads to the reduced row echelon form

$$\left(\begin{array}{ccccc|c} 1 & 5 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Find all solutions to the system $A\mathbf{x} = \mathbf{b}$.

$x + 5y \quad - v = -1$
$z \quad + v = 3$
$w + 2v = 8$
y, v free

$$\begin{pmatrix} x \\ y \\ z \\ w \\ v \end{pmatrix} = \begin{pmatrix} -1-5y+v \\ y \\ 3-v \\ 8-2v \\ v \end{pmatrix} = \left\{ \begin{pmatrix} -1 \\ 0 \\ 3 \\ 8 \\ 0 \end{pmatrix} + y \begin{pmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + v \begin{pmatrix} 1 \\ 0 \\ -1 \\ -2 \\ 1 \end{pmatrix} ; y, v \in \mathbb{R} \right\}$$

2. Complete the following statement:

The solutions to $A\mathbf{x} = \mathbf{b}$ are given by first taking the span of the vector(s) $(-5,1,0,0,0)$ and $(1,0,-1,-2,1)$

and then translating this span by the vector $(-1,0,3,8,0)$.

3. If for a new vector $\mathbf{c}$ we attempt to solve $A\mathbf{x} = \mathbf{c}$, must it have solutions? may it have solutions? Explain briefly. It may have solutions, but also may not. We know the reduced echelon form of A has a row of zeros, so it depends on whether the corresponding entry in the last column of r.r.e.f of $(A|c)$ has a zero or not.

4. What are the solutions of the homogeneous system $A\mathbf{x} = \mathbf{0}$?

$$y \begin{pmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + v \begin{pmatrix} 1 \\ 0 \\ -1 \\ -2 \\ 1 \end{pmatrix} \quad \text{where } y, v \in \mathbb{R}$$