Final Exam Name:__________________

Math 202 F01 12/15/05

1. An object fills the solid region between the graph of $z = 16 - x^2 - y^2$ and the plane $z = 0$. The density at any point is given by $f(x, y, z) = 2z$. Here x, y, z are measured in cm, and f is in g/cm^3.

   (a) (3 pts.) Sketch the object.

   (b) (10 pts.) Compute the mass of the object.

2. (6 pts.) For points near $(4, 1)$, give a linear approximation to the function

$$f(x, y) = xy^2 - 3\sqrt{xy}.$$

3. (6 pts.) What is the directional derivative of $f(x, y, z) = 2xy - z^2$ at the point $(1, 2, 1)$ in the direction toward the origin?

4. Consider the vector field $F(x, y, z) = (2xy + z^2, x^2 + z - 1, 2 + y + 2xz)$.

   (a) (7 pts.) Either find a potential function for F, or show one does not exist.

   (b) (7 pts.) Supposing F represents a force field, compute the amount of work done by F on a particle moving along the path

   $$\mathbf{r}(t) = (1 + t, t^2, t^3 - 4t), \quad 0 \leq t \leq 2.$$

   (You may find it helpful to use your answer to part (a).)

5. (15 pts. — 3 pts. each) Consider the point $\mathbf{p} = (1, 2, -1)$, and the two vectors $\mathbf{v} = (1, 0, -1)$, $\mathbf{w} = (2, -2, 2)$.

(a) Give a parameterization of the line through $\mathbf{p}$ with direction $\mathbf{w}$.

(b) Find a vector orthogonal to both $\mathbf{v}$ and $\mathbf{w}$.

(c) Give an equation of the plane through $\mathbf{p}$ containing the directions $\mathbf{v}, \mathbf{w}$.

(d) Give a parameterization of the plane through $\mathbf{p}$ containing the directions $\mathbf{v}, \mathbf{w}$.

(e) Find the angle between $\mathbf{v}$ and $\mathbf{w}$.

6. (10 pts.) A surface S is parameterized by

$$\mathbf{r}(u, v) = (u \cos v, u, u \sin v), \quad 1 \leq u \leq 2,\ 0 \leq v \leq \pi$$

Set up an integral to compute the flux through S of the vector field $F(x, y, z) = (x, 1, y)$. *Do not evaluate the integral, but leave it in a form where that is all that remains to be done.*

7. (10 pts.) Use Green's theorem to evaluate the integral

$$\oint_C x^2 y^2 \, dx - 2x^3 y \, dy$$

where C is the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 3)$.

8. (10 pts.) A wire, with uniform density, is shaped like a semicircle around the origin, going from $(2, 0)$ to $(-2, 0)$ in the plane. Find the coordinates of the centroid of the wire (i.e., of the center of mass).

9. (16 pts. — 2 pts. each) The following may or may not be correct. If they are, mark them 'True.' If they are not, give corrected versions.

   (a) In spherical coordinates, $dV = \rho^2 \cos\phi \, d\rho \, d\phi \, d\theta$.

   (b) In cylindrical coordinates, $dV = r \, dz \, d\theta \, dr$.

   (c) The area of a parallelogram with sides given by vectors $\mathbf{v}$ and $\mathbf{w}$ is $\mathbf{v} \cdot \mathbf{w}$.

(d) If $\nabla f(1,-2) = (0,0)$, then f must have a local maximum or a minimum at $(1,-2)$.

(e) If $-C$ denotes 'C backwards,' then $\int_C F \cdot d\mathbf{r} = \int_{-C} F \cdot d\mathbf{r}$.

(f) If $-C$ denotes 'C backwards,' then $\int_C f\, ds = \int_{-C} f\, ds$.

(g) The main reason its useful to know how to compute integrals is to be able to find areas.

(h) If div $F(1,-1,2) = (1,0,0)$, then the vector field F has a net flow out from the point $(1,-1,2)$ in the direction given by $(1,0,0)$.