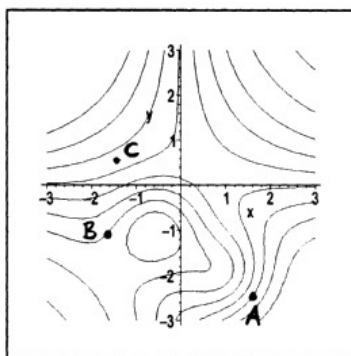


1. (10 pts.) A plot of level curves of a function  $z = f(x, y)$  is given:



- (a) At the points A, B, C, draw the gradient for  $f$ . You should be able to indicate the correct direction, and correct *relative* lengths of the three.
- (b) Mark with an 'X' all points where the gradient is  $\vec{0}$
2. (16 pts.) A metal plate is shaped like the region between the three lines  $y = \pm 2x$  and  $x = 5$ , where  $x$  and  $y$  are measured in  $m$ . The density of the plate is given by  $\rho(x, y) = 1+x \text{ kg/m}^2$ . Give an expression involving integrals for the center of mass of the plate. (Do NOT evaluate any integrals, but leave your answer in a form where that is all that remains to be done.)

3. (14 pts.) Give an equation of the tangent plane to the surface

$$xy + yz^2 + zx = -1$$

at the point  $(1, 0, -1)$ .

4. The temperature at a point  $(x, y)$  is given by

$$T = \frac{y}{1 + x^2} + z^2.$$

- (a) (10 pts.) Compute the directional derivative of temperature at the point  $(2, 1, 1)$  in the direction towards  $(3, 2, 1)$ .

- (b) (6 pts.) At the point  $(2, 1, 1)$ , in what direction should you travel in order to get the fastest *increase* in temperature?

5. (28 pts. – 14 pts. each) Evaluate the following integrals by first following the suggestions.

(a)  $\int_0^3 \int_{x^2}^9 x \cos(y^2) dy dx$  (Reverse the order of integration.)

(b)  $\int_0^3 \int_0^{\sqrt{9-x^2}} \sqrt{(10-x^2-y^2)} dy dx$  (Use a different coordinate system.)

6. (16 pts.) Find the maximum of the function  $f(x, y, z) = 8x - 4z$ , subject to the constraint that  $x^2 + 10y^2 + z^2 = 5$