Exam 1 Name:_____

Math 202 F01 10/05/05

1. (25 pts. — 5 pts. each) Three points, with coordinates

$$A = (1, -1, 0), \quad B = (1, 2, 1), \quad C = (-1, 0, 1),$$

are the vertices of a triangle in 3-dimensional space.

(a) What is the length of the side joining A and B?

(b) Give a *unit* normal vector to the plane containing the triangle.

(c) Give an equation of the plane containing the triangle.

(d) What is the angle formed by the sides meeting at A? (You may leave your answer in a form involving inverse trigonometric functions.)

(e) What is the area of the triangle?

2. (10 pts.) An object at the origin in the plane is acted on by a force F of magnitude $15N$ that attempts to push it toward the point $(-3, 4)$. However, the object is constrained so that it can only move along the line from the origin toward $(-3, 3)$. Find F, and then the vector projection of F onto the direction the object can move.

3. (16 pts. — 8 pts. each) Sketch graphs of the following, in 3-dimensions. You may use whatever approach you like, and need not produce high quality drawings, but it must be clear from your sketches that you understand the correct shape.

   (a) $z = y^3$

   (b) $z = \dfrac{1}{x^2 + y^2}$

4. (8 pts. — 4 pts. each) In Cartesian coordinates, $\mathbf{p} = (0, -2, 2)$. Express $\mathbf{p}$ in:

   (a) cylindrical coordinates

   (b) spherical coordinates

5. An object moves along a trajectory so that its position, as a function of time, is given by

$$\mathbf{r}(t) = (t^2, 2t, \ln(t)).$$

   (a) (5 pts.) At what speed is it traveling at time $t = 2$?

   (b) (8 pts.) What is the length of its trajectory between times $t = 1$ and $t = 2$?

   (c) (6 pts.) Give a parameterization of the line tangent to the trajectory at $\mathbf{r}(2)$.

6. (10 pts.) In the plane, a particle moves so that it has constant acceleration $\mathbf{a}(t) = 2\mathbf{j}\ m/s^2$.

   At $t = 0$ it has velocity $\mathbf{v}(0) = -1\mathbf{i} + 1\mathbf{j}\ m/s$,

   and at time $t = 1$ its position is $\mathbf{r}(1) = 2\mathbf{i} - 1\mathbf{j}\ m$.

   Give a formula for its position, $\mathbf{r}(t)$, at all times.

7. The temperature (in $^\circ C$) at each point (x, y), $-2 \leq x, y, \leq 2$, on a 4×4 metal plate is given by $T(x, y) = 15 + x^2 - y$.

   (a) (9 pts.) Draw a contour plot of T that shows the level curves (i.e, isotherms) where $T = 14,\ 15$, and 16.

   (b) (3 pts.) Using only these three isotherms as guides, on your plot above mark the point(s) on the metal plate (with $-2 \leq x, y \leq 2$) that you suspect are the hottest with an 'H', and those that you think are the coolest with a 'C'.