Assignment # 5 (*with revised* P13)

Due Monday 16 March at start of class

Exercise 6.1 in Tref & Bau.

Exercise 6.3.

Exercise 6.4.

Exercise 7.3.

P12. Write Algorithm 7.1, page 51, as a MATLAB|OCTAVE|PYLAB function, `[Q,R]=clgs(A)`. You might want to mimic the *style*, though of course not the content, of `mgs.m` posted at

`http://www.dms.uaf.edu/~bueler/mgs.m`

We will get to the modified Gram-Schmidt algorithm later, but it computes the same thing as the classical version, and as M|O|P's built-in "QR": `[Q,R]=mgs(A)` and `[Q,R]=qr(A)`.

On page 51 TREF & BAU claim classical Gram-Schmidt is unstable. It turns out to be a rather subtle instability (compared to some more obvious examples later, certainly). Demonstrate this instability by following "Experiment 2" on pages 65–66. In particular, reproduce figure 9.1. The labels on figure 9.1 are not the issue, of course, but in producing the figure you may find it useful to know `eps = 2.2204e-16`.

P13. *REVISED!: There were typos in the columns of A, and compassion suggests a bit less work, so A has 4 columns.* First, read pages 52–54 on "When Vectors become Continuous Functions". Then do an analogous calculation for the interval $[0,\infty)$, as follows:

Suppose the inner product of f and g is the integral

$$(f,g) = \int_0^\infty \overline{f(x)}g(x)\,dx.$$

Consider the "$[0,\infty)\times 4$" matrix

$$A = \left[\begin{array}{c|c|c|c} & & & \\ e^{-x} & x\,e^{-x} & x^2\,e^{-x} & x^3\,e^{-x} \\ & & & \end{array} \right].$$

That is, consider $x^j e^{-x}$, for $j = 0,\dots,3$, which are functions on $[0,\infty)$, as the columns of a matrix. Find Q, R so that $A = QR$. (*That is, do Gram-Schmidt exactly by hand. But record the coefficients r_{ij} as you go, in the appropriate manner, so that the columns of A can be recovered from R and the orthogonal polynomials, the columns of Q. You should know the dimensions of R in advance.*)