Assignment # 3

Due Wednesday 18 February at start of class

"M|O|P" = MATLAB|OCTAVE|PYLAB.

Exercise 9.1. Page 68 of TREF & BAU. Do it in M|O|P. Note that the "six-line MATLAB program of Experiment 1" is `legendre.m` (and `legendre.py`) online at the course webpage, so you don't even need to type it in.

Exercise 3.3. TREF & BAU

Exercise 3.5. TREF & BAU

P8. Let

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 5 & 8 \\ 5 & 8 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 8 & 1 & -1 \\ 3 & 5 & 7 \\ 4 & 19 & 1 \end{bmatrix}.$$

(a) Using formulas on page 21, determine $\|\cdot\|_1$ and $\|\cdot\|_\infty$ of A and B.

(b) Use M|O|P to determine the norms $\|\cdot\|_1$, $\|\cdot\|_2$, and $\|\cdot\|_\infty$ of A and B. Also determine $\rho(A)$, $\rho(B)$ with M|O|P; see exercise 3.2 in TREF & BAU for the definition of "$\rho(A)$". Are the results of exercise 3.2 true for A and B?

(c) Find a 3×3 matrix C for which $\|C\|_1$ is more than 100 times larger than $\rho(C)$.

(Extra Credit) Estimate $\|A\|_4$, $\|B\|_4$, and put your method in a M|O|P function.

P9. I wrote a three line, 79 character, MATLAB/OCTAVE program to produce the following picture of the unit ball in $\mathbb{R}^2$ using the norm $\|\cdot\|_4$. You?

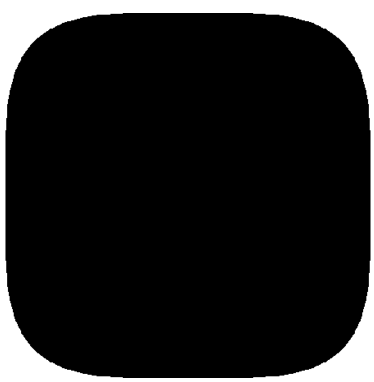

FIGURE 1. Conference table.

P10. In discretizing a differential equation, I constructed an $m \times m$ matrix A with entries

$$a_{ij} = \begin{cases} 2 + 9(m+1)^{-2}\cos(j/(m+1)) & \text{if } i = j, \\ -1 & \text{if } i = j+1 \qquad (j = 1, \dots, m-1), \\ -1 & \text{if } i = j-1 \qquad (j = 2, \dots, m), \\ 0 & \text{otherwise.} \end{cases}$$

This question will exercise M|O|P manipulations of such a matrix. You do not need to know anything about DEs to do it. The goal is to learn the "right" way to use M|O|P here, i.e. to find the tools that are suited to this common kind of matrix. In particular, this matrix has strong structure, with lots of zeros. On the other hand, it is nontrivial enough so that one actually has to do something numerical, as below, to solve the DE which generated it.

Please do not print out the entries of any of the vectors and matrices in your answer. I will never want to see a printed array of 40×40 numbers, or even a 40×1 array. Find other ways to communicate your answers, and to make it clear you succeeded.

(a) Let $m = 40$. Use M|O|P to enter A in an efficient manner.

(b) Use M|O|P to find A^{-1}. Use `spy` on A and A^{-1} and describe the result in words.

(c) For $b \in \mathbb{C}^m$ with entries $b_j = j(m+1)^{-3}$, use M|O|P in the correct and briefest manner to compute $c = Ab$ and to solve $Av = b$ for v. Report $\|b\|_\infty$, $\|c\|_\infty$, $\|v\|_\infty$.

(d) Use M|O|P to find the largest four eigenvalues of A and the smallest four eigenvalues of A.

(e) Put steps **(a)**, **(c)**, **(d)** in a M|O|P script or function which allows the setting of an arbitrary value for m; no `for` loops or other nontrivial programming constructs are necessary. Then repeat all steps with $m = 4000$. Try step **(b)** with $m = 4000$.

(Extra credit) Run your script/function with $m = 10^5$, if possible. If this is possible, explain what tools made it so.

(More extra credit) You need not know anything about DEs to do this question, *until now*: Explain why some of the results in the $m = 40, 4000$, and 10^5 cases—or just the first two if the largest m was not implementable—are close to each other. Explain why other results are not close to each other for different m.