

Name: _____

Math 630 (*Practical*) Advanced Linear Algebra (Bueler)

31 October, 2003

Midterm Quiz on Fundamentals

(40 *points possible*)

1. (5 *pts*) Give the definition: $\|\cdot\|$ is an *induced norm* (on matrices).

2. (5 *pts*) Show $\text{range}(I - P) = \text{null}(P)$ if P is a projector.

3. (5 pts) Show that if P is a projector and $P^* = P$ then $\text{range}(P) \perp \text{null}(P)$.
(By definition, $V \perp W$ if $x^*y = 0$ for all $x \in V$ and all $y \in W$.)

4. (5 pts) Suppose A is a 3 by 3 matrix with an SVD

$$A = U \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} V^*.$$

Suppose in addition that $u_1 = -v_1$, $u_2 = v_2$, and $u_3 = -v_3$. What are the eigenvalues of A ? Prove it.

5. (5 pts) By definition, the *row rank* of a matrix A is the dimension of the row space of A (that is, the span of the rows) and the *column rank* of a matrix A is the dimension of the column space of A (that is, the span of the columns).

Show that for any matrix A the row rank equals the column rank. (*Hint: SVD.*)

6. (5 pts) Suppose A has QR factorization

$$Q = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & -1 \end{bmatrix}.$$

Suppose $b = [2\sqrt{2} \quad -\sqrt{2} \quad 3]^*$. Solve $Ax = b$ for x .
(Hint: Easily doable by hand. Don't compute A !)

7. (5 pts) Suppose A is an $m \times n$ matrix. Printed below is the classical Gram-Schmidt iteration with lines containing floating point computations indicated by boxed numbers:

Count operations in this algorithm and show that it requires $\sim K mn^2$ flops for some a . In particular, explain which of lines $\boxed{1}, \dots, \boxed{4}$ contributes to this leading order estimate and then find, with explanation, the constant K .

(Recall that “number of flops $\sim K mn^2$ ” means

$$\lim_{m,n \rightarrow \infty} \frac{\text{number of flops}}{K mn^2} = 1.)$$

8. (5 pts) Suppose $A = U\Sigma V^*$ is an SVD of a $m \times n$ matrix. Clearly state an important minimization problem, associated to A , whose solution is the rank one matrix $\sigma_1 u_1 v_1^*$.