

Assignment # 2

Due Friday 9/19 at start of class

Relatively(!) easy proofs:

Exercise 2.1. (Page 15, in Lecture 2 in TREFETHEN. I suppose from now on that you can find exercises in the text easily enough.)

Exercise 2.2.

Exercise 2.3.

A MATLAB introduction:

II. In discretizing a PDE¹ I happen to construct an $m \times m$ matrix A with entries

$$a_{ij} = \begin{cases} 2 & \text{if } i = j, j = 1, \dots, m, \\ -1 & \text{if } i = j + 1, j = 1, \dots, m - 1, \\ -1 & \text{if } i = j - 1, j = 2, \dots, m, \\ 0 & \text{otherwise.} \end{cases}$$

(a) For $m = 40$, use MATLAB to enter this matrix in an efficient manner.

(b) Use MATLAB to find A^{-1} . Do not show me this matrix, but describe any patterns you see.² Use `spy` on A and A^{-1} ; describe the result in words.

(c) For $b = [1 \ 2 \ \dots \ 40]^*$ use MATLAB in the correct and briefest manner to solve $Av = b$ for v . Report v without using too much paper.

(d) Find the eigenvalues of A and A^{-1} (don't report them). How close are these numbers to satisfying $(\text{eigs of } A^{-1}) = \frac{1}{(\text{eigs of } A)}$? If there is an error, what do you think is the source of the error?

Exercise 9.1. (page 68)

Do one of the following (the first one, a proof, is a little easier for me):

Exercise 2.5. (Correction: Part (a) should say “Show by using Exercise 2.3 that ...”)

Exercise 9.3. Do part (a). Then answer the question (a question in part (b)): What is the mathematically exact rank of A ?

Extra Credit:

III (Extra Credit). First, read exercise 2.7. (To gain familiarity with Hadamard matrices, sketch for yourself how to do this exercise.) Find Hadamard matrices of dimensions 1, 2, 4, 8, 12, 16, 20, 24. [*I do not know how hard this exercise is!*]

¹“PDE” = partial differential equation; you need not know anything about PDEs to do this question!

²I don't need to see 40² numbers *ever* in this class—I want you to actually communicate to me!