Assignment #8

Due *Friday April 13, 2007.*

1. Read sections 4.6, 4.7, 4.9, 4.11, 4.12, 4.13.

2. [*Easy.*] The Lax-Wendroff method can be written several ways. **Show** that (4.49) in Morton & Mayers 2nd ed. reduces to the first form of the method (4.36) if a is constant (i.e. $f(u) = au$).

One must be a bit careful in going back and forth from the form of a first order equation where the velocity a is clear (equation (4.47)) and the form where the flux f is clear (equation (4.46)). In particular, if $a = a(x)$ then the equation $u_t + a(x)u_x = 0$ generally can't be written in a homogeneous form like (4.46), namely

$$u_t + \big(f(x,u)\big)_x = 0. \tag{1}$$

If $a = a(x)$ one can, however, write $u_t + a(x)u_x = 0$ in the form of equation (1) with the right hand side changed by an frequently harmless term. **Write** this equation; I'll denote it (1'). **Describe** under what circumstances you think the right hand side of (1') is harmless (i.e. is easy to deal with numerically). **Explain** how to apply Lax-Wendroff to (1') using a slight modification of (4.49).

2. Figure 4.8 in Morton & Mayers 2nd ed. shows an exact and approximate solution to the linear advection equation (4.33) with initial-boundary condition (4.34b) and initial data (4.45). The approximate solution in figure 4.8 is by Lax-Wendroff.

(a) Reproduce this figure. That is, write a program using Lax-Wendroff, in this case equation (4.44), to reproduce it. You will need to enforce CFL. Note you will use the exact characteristics (4.35b) to plot the exact solution.

(b) For comparison purposes, produce the corresponding figure 4.8 but using the *upwind method*. Again you will need to enforce CFL; the exact solution is the same as in **(a)**. Once you have produced the figure, compare it to figure 4.8 in several sentences using the ideas in the text (i.e. truncation error, maximum principle, damping of high frequency modes, relative phase error).

(c) Now redo each of the figures in parts **(a)** and **(b)** but *violating* CFL. In particular, choose the times step Δt_n at time t_n by the rule

$$\frac{(\max_j |a(x_j, t_n)|)\,\Delta t_n}{\Delta x} = 2.$$

Comment on the results.

3. Show that equations (4.65) in Morton & Mayers 2nd ed. reduce to (4.59) in the case $\mathbf{f} = A\mathbf{u}$ with A constant. Furthermore, show that if there is only a single equation with $f = au$, a constant, then both (4.65) and (4.59) reduce to (4.36).