Assignment #7

Due *Wednesday April 4, 2007.*

1. Read sections 4.1, 4.2, 4.3, 4.4, 4.5.

2. ***(a)*** Solve

$$u_t + (xt)u_x = 0, \qquad u(x,0) = \cos x,$$

for $x \in \mathbb{R}$ and $t \geq 0$, by hand. That is, apply the method of characteristics. Check your answer $u(x,t)$ by substitution into the PDE.
(b) Sketch the characteristics in the (x,t) plane.
(c) Solve

$$u_t + (xt)u_x = 1, \qquad u(x,0) = \cos x,$$

for $x \in \mathbb{R}$, by hand. [*Hint: There is no need to redo what you did in part* **(a)***. Simply add what you need to solve this new problem.*]

3. ***(a)*** Write a MATLAB program which numerically approximates $u(x,t)$ solving

$$u_t + (xt)u_x = 0, \qquad u(1,t) = \cos(1), \qquad u(x,0) = \cos x,$$

for $1 \leq x \leq 3$ and $0 \leq t \leq 1$, using the upwind method. Using $\Delta x = 0.1$, show the approximate solution at $t = 1$. (That is, show the approximation to $u(x,1)$ for $1 \leq x \leq 3$.) Note that the CFL condition determines the time step Δt from the given values for Δx; you can do this adaptively or by fixed time steps.
(b) Verify your solution at $t = 1$ using the exact solution. (To find the exact solution, use the result from problem **2*(a)*** and add what is needed.) In particular, find the rate of convergence of the worst case error at the final time (namely, $t = 1$) for these grid intervals: $\Delta x = 0.4, 0.2, 0.1, 0.05, 0.02, 0.01, 0.005, 0.002, 0.001$. Estimate the rate of convergence as a power of Δx.

4. Exercise 4.1 in MORTON & MAYERS 2ND ED. (*Note that you will not* have *to write any code for this one. On the other hand, it may be helpful to modify the program in the previous problem to do this problem, just so that you can see how it acts and think about your result.*)