Assignment #6

Due *Friday March 23, 2007.*

1. Read sections 3.1, 4.1, and 4.2 of MORTON & MAYERS, 2ND ED.

2. Apply an adaptive time-stepping explicit method to the following nonlinear heat problem:

$$u_t = \left(e^{-5\arctan u}\right) u_{xx} - 1, \qquad u(0,t) = 0, \quad u(1,t) = 0, \quad u(x,0) = x(1-x).$$

That is, incorporate the condition (2.171) into your program to choose each time step. Approximate $u(x,t)$ at $t_f = 0.5$. Comment on how the behavior of the solution affects the time steps. In fact, plot the time steps as a function of the time for the grids $\Delta x = 0.05$ and $\Delta x = 0.01$.

[*Notes*: Use `exp` and `atan` in MATLAB for e^x and $\arctan x$. Also, I found that $\Delta x = 0.01$ is a mesh size which produces a reasonable execution time. Nonetheless you might try finer meshes so that you are confident you are seeing a close approximation of the solution. Truthfully, I have no idea what the formula for the exact solution might be. I am, however, completely confident there is only one such solution and that it is smooth.]

3. This is really a second part for the previous problem. Apply upwinding to the advection term in

$$u_t = \left(e^{-5\arctan u}\right) u_{xx} - \left(\frac{8t\cos(\pi x)}{1+8t}\right) u_x - 1$$

on the same domain $(x,t) \in [0,1] \times [0,0.5]$ and with the same initial and boundary conditions as in the previous problem. In particular, add condition (2.149) to your adaptive time-stepping scheme. Is this condition ever active, that is, does (2.149) ever determine the time step or is the time step always determined by (2.171)?

4. Exercise 3.1 (page 83 of MORTON & MAYERS, 2ND ED).