Assignment #3

Due *Friday, 9 February 2007.*

1. Read sections 2.7 and 2.8 (pages 22–26) in MORTON & MAYERS, 2ND ED.

Also browse section 2.9, but note that we will skip it, except for the discussion of formulas (2.72), (2.73), and (2.74). (*Section 2.9 discusses the "Thomas algorithm" for solving tridiagonal systems. That discussion is tangential to our main purpose, which is approximating the continuum system rather than solving the finite-dimensional approximation. In particular,* MATLAB *already does an excellent job of solving tridiagonal systems; it has the Thomas algorithm built-in.*)

2. Suppose that $f(x)$ has a continuous third derivative which is bounded: there is M so that $|f'''(x)| \le M$ for all x of interest. By considering Taylor's theorem with remainder on $f(x+2\Delta x)$ and $f(x+\Delta x)$, show carefully that

$$\left| f''(x) - \frac{f(x+2\Delta x) - 2f(x+\Delta x) + f(x)}{\Delta x^2} \right| \le 2M\Delta x.$$

Explain why this inequality describes a less-good finite difference approximation of the second derivative than the centered formula given in class. Also explain why one cannot prove a formula of the form

$$\text{“} \quad f''(x) = \frac{f(x+2\Delta x) - 2f(x+\Delta x) + f(x)}{\Delta x^2} + Cf'''(\xi)\Delta x \quad \text{”}$$

for any constant C.

3. Exercise 2.3 in MORTON & MAYERS, 2ND ED (page 58).

4. Reproduce figure 2.4 in MORTON & MAYERS, 2ND ED (page 19). (*The point here is* not *that you still need to practice using* MATLAB*, though that may be true! Rather I want you to practice evaluating actual errors when an exact solution is known, and I want you to see that this is a good way to display them.*)