Assignment #2

Due *Friday, 2 February 2007.*

1. Again read sections 2.2 through 2.6 (pages 6–19) in MORTON & MAYERS, 2ND ED..

2. Do exercise 2.1 in MORTON & MAYERS, 2ND ED. (pages 56–57).

3. Reproduce figure 2.2 in MORTON & MAYERS, 2ND ED. (page 13) using MATLAB. Note that this figure includes both the exact solution from the Fourier series (2.11) *and* the approximate solution from the explicit scheme (2.19) of the problem (2.7)–(2.9) using initial condition (2.24).

Here is a *suggested solution procedure to reproduce figure 2.2*: First write a program that will plot a truncation of the exact solution (2.11), with the appropriate constants. (See the previous problem.) Then write a program to compute the explicit approximation. This second program will step forward using (2.19) assuming a particular value for Δt; you can fix $\Delta x = 0.05$. Then display both the exact and approximate solutions as in figure 2.2, possibly by writing a third program. Plot using MATLAB reasonably carefully to reproduce all the essential features of figure 2.2. For instance, use `subplot` and also `hold`, and the plot appearance option in the command `plot(x,y,'.-')` when plotting the approximate solution.

4. *This is a fairly short* MATLAB *exercise*: Clean up `bob.m`. In particular,

`http://www.dms.uaf.edu/~bueler/bob.m`

is a MATLAB m-file with three "`for`" loops. You can remove two of them. The first loop, which computes the initial value, can be removed by use of the entry-by-entry array multiplication command "`.*`". The inner loop over "`j`" can be removed by using colon notation. Next make `bob` into a function which takes as input `J`, `N`, and `tf`, and which returns the final state. Also have `bob` print out the value of "`mu`". Finally, by choosing increasing values of `J` and `N`, try to approximate $u(x,t)$ at $x = 0.3$ and $t = 0.1$ to five digits. List a few results so that there is evidence that you have such accuracy.

5. (Extra Credit; pretty hard). Find the separation of variables solution to

$$u_t = (1+2x)u_{xx}$$
$$u(0,t) = 0 \quad \text{and} \quad u(1,t) = 0$$
$$u(x,0) = x(1-x)$$

It will involve special functions; the main task is to find them and their relevant properties. Then use the infinite series solution to approximate and plot (e.g. using MATLAB), the graphs of $u(x,0)$, $u(x,0.01)$, $u(x,0.1)$, and $u(x,1.0)$ on common axes. Regarding your truncated infinite series as an exact solution, evaluate the error made by `bob.m` in computing $u(x,0.1)$. What is a good way to report that error? What issues arise in using your truncated infinite series as an exact solution, if any?