Assignment #1

Due *Friday, 26 January 2007.*

1. Read *lightly* the prefaces and introduction of the textbook MORTON & MAYERS. Read *seriously* the "Numerical Analysis" handout by Trefethen. Find textbooks on calculus and ordinary differential equations (ODEs) which will explain these two topics, respectively: Taylor's theorem with remainder formula and the solution of linear homogeneous constant-coefficient ODEs. Read *seriously* subsections 2.1, 2.2, 2.3, 2.4, and 2.5 of MORTON & MAYERS.

2. Solve

$$2y^{(4)} + 11y''' + 16y'' + y' - 6y = 0, \tag{1a}$$

$$y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 2, \quad y'''(0) = 3. \tag{1b}$$

In particular, compute $y(10)$. This is an *ordinary differential equation initial value problem* or *ODE IVP*.

(In this case, the polynomial equation which will likely appear in your solution method can be solved by hand, but MATLAB can do it too. Also MATLAB can be used to solve any linear (algebraic) systems which appear. Show a couple of lines of input and output when you use MATLAB for such small tasks. Do not use MATLAB's built-in ODE solvers `ode45,ode23s,...` in this problem.)

3. Calculate $\sqrt{16.1}$ to five digits without any computing machinery except a pen(cil). Prove that your answer is correct to five digits, again without any computing machinery!

4. Assume f has continuous first and second derivatives. Derive the midpoint-rule-with-remainder formula

$$\int_{-a}^{a} f(x)\,dx = 2af(0) + \frac{1}{3}a^3 f''(\nu)$$

for some (unknown) $-a \le \nu \le a$. [*Hint*: $f(x) = f(0) + f'(0)x + (1/2)f''(\xi)x^2$ where $\xi = \xi(x)$ is between 0 and x.] How accurate is the midpoint rule on the integral $\int_{-0.1}^{0.1} e^x\,dx$? Use two sentences to explain the meaning of this formula to the layperson.

5. Solve

$$y''' + 5y'' - 5y' - y = 0, \quad y(1) = 0, \quad y'(1) = 0, \quad y''(1) = 2 \tag{2}$$

to find $y(3)$. (Do not a MATLAB's built-in ODE solver yet. Note that you will *not* be able to solve the polynomial equation which appears by hand, probably, but that MATLAB has excellent numerical methods for doing this; find one!)

6. Use MATLAB's `ode45` to solve initial value problem (2) and find $y(3)$. Equation (2) can and must be written as a first order system before using the built-in solver.

7. Using Euler's method for approximately solving ODEs, write your own MATLAB program (either script or function) to solve initial value problem (2) to find $y(3)$. Use a few step sizes, decreasing as needed, so that any reasonable observer would agree that you have four digit accuracy. (Even if the observer has not already done problem **5** or **6** above, that is.)