Assignment #9

Due *Monday, 18 April, 2005.*

1. Read sections 6.1, 6.2, 6.3, 3.1, 3.2.

2. On the course webpage `www.cs.uaf.edu/~bueler/Math615S05.htm` you will find a MATLAB function called `potential.m` and a figure `poterrs.jpg`. Type `help potential`, run `potential.m` at least once, and look at `poterrs.jpg`.

Now, `poterrs.jpg` rather clearly shows that the error from `potential` is $O(\Delta x^2)$; this is also the result of the analysis in section 6.2—see equation (6.19). It is not, however, obvious how fast `potential` is as a function of its first argument `J`, the mesh parameter. This is because the speed depends on the speed of the sparse matrix solution "`w=A\b`."

Explore this question by timing `potential` with a range of `J` values for some particular boundary conditions. (Note that the exact solution need not be known for this timing question.) Supposing

$$\text{(execution time)} \approx c\, \mathtt{J}^p,$$

for some constant c, estimate p. Produce a graph like `poterrs`, with the x-axis labeled "`J`" and the y-axis labeled "`execution time`." Then give an argument for why this p value arose, especially in terms of the solution of the linear algebra problem. (I claimed in class that it is clear that $2 \le J \le 6$; you may want to start by reproducing this explanation.) Finally, answer the question: how long would your computer take to run `potential` with input $\mathtt{J} = 1000$? with $\mathtt{J} = 10^5$?

3. Set up a finite difference method for the two point boundary value problem

$$u'' + (\sin x)u' - 5u = x^2 - 2, \qquad u(0) = 1, \quad u'(1) = 0.$$

Compute a good approximate solution $u(x)$ for an appropriately large number of mesh points. Check your program by constructing a manufactured exact solution of a nearby problem and comparing the result of your program to that exact solution; in particular, demonstrate convergence to the manufactured solution. Finally, comment on how you would modify your program if the problem were nonlinear—in particular consider the case: $u'' + f(x)u' + g(x)u^2 = h(x)$.