Assignment #7

Due *Monday March 28, 2005.*

1. Read sections 4.3, 4.4, 4.5.

2. Apply the "method of manufactured solutions" to find an exact solution to

$$u_t = \left(e^{-30 \arctan u}\right) u_{xx} + f(x,t), \qquad u(0,t) = 0, \quad u(1,t) = 0.$$

In particular, find $f(x,t)$ so that

$$u(x,t) = e^{-t/3} \sin(\pi x)$$

is the exact solution. Identify the initial condition $u_0(x) = u(x,0)$. [*Hint*: All of this is very close to what was done in class.]

Now *verify* the adaptive explicit method which solved exercise **3** on Assignment # 6 using this exact solution. That is, modify that code to allow for $f(x,t)$ and then compare the computed U_j^n to the exact $u(x,t)$ at (in particular) $t_A = 1.0$ and $t_B = 5.0$. Demonstrate numerically that as the grid is refined (i.e. as $\Delta x \to 0$; the adaptive scheme will then force $\Delta t \to 0$) the approximate solution approaches the exact solution at $t = t_A, t_B$. I recommend using $\Delta x = 0.01$ as your *finest* grid. Produce a good graph or two which demonstrates this evidence for convergence.

[*You may modify the code* `nonlinheat.m` *which appears on the class webpage if you prefer it over the code you wrote for Assignment # 6.*]

3. ***(a)*** Solve

$$u_t + (xt)u_x = 0, \qquad u(x,0) = \cos x$$

by hand. That is, apply the method of characteristics. Check your answer.

(b) Sketch the characteristics in the (t,x) plane.

(c) Solve

$$u_t + (xt)u_x = 1, \qquad u(x,0) = \cos x$$

by hand. [*Hint*: There is no need to redo what you did in part ***(a)***; simply add what you need to solve this new problem.]

4. Exercise 4.1 in Morton & Mayers.