Assignment #5

Due *Monday March 7, 2005.*

1. Read sections 2.13, 2.14, 2.15, and 2.16.

2. Exercise 2.6 (page 57).

3. Show that the explicit scheme

$$\frac{U_j^{n+1} - U_j^n}{\Delta t} = \frac{(U_{j+1}^n - U_j^n)p_{j+1/2} - (U_j^n - U_{j-1}^n)p_{j-1/2}}{\Delta x^2}$$

for the differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(p(x)\frac{\partial u}{\partial x}\right)$$

is consistent assuming $p(x)$ has continuous derivative. Note $x_{j+1/2} = (x_j + x_{j+1})/2$ and $p_{j+1/2} = p(x_{j+1/2})$.

Hint: You seek the leading terms in the truncation error. Use Taylor's theorem to get

$$\begin{aligned} &p(x+\epsilon)\left[u(x+\Delta,t) - u(x,t)\right] \\ &\quad = (p(x) + p'(\xi_1)\epsilon)\left[u_x(x,t)\Delta + \frac{1}{2}u_{xx}(x,t)\Delta^2 + \frac{1}{6}u_{xxx}(x,t)\Delta^3 + \frac{1}{24}u_{xxxx}(\xi_2,t)\Delta^4\right]. \end{aligned}$$

Now use this to compute

$$p(x+\Delta x/2)\left[u(x+\Delta x,t) - u(x,t)\right] - p(x-\Delta x/2)\left[u(x,t) - u(x-\Delta x,t)\right],$$

and consider $\lim_{\Delta x\to 0}$ of this quantity divided by Δx^2. *This easier exercise replaces Exercise 2.7, page 57.*

4. Exercise 2.9 (pages 57–58).