Assignment #1

Due *Monday, 31 January 2005.*

1. Read *lightly* the preface and pages 1 through 9 of the textbook Morton & Mayers.

2. Solve

(1) $$y''' + y'' + y' + y = 0, \quad y(1) = 0, \quad y'(1) = 1, \quad y''(1) = 2.$$

In particular, compute $y(10)$. [*Hint*: $m^3+m^2+m+1 = (m^2+1)(m+1)$. Also: Matlab handles matrices well!]

3. Calculate $\sqrt{16.1}$ to five digits without any computing machinery. Prove that your answer is correct to the given digits, again without any computing machinery!

4. Assume f has continuous first and second derivatives. Derive the midpoint-rule-with-remainder formula

$$\int_{-a}^{a} f(x)\,dx = 2af(0) + \frac{1}{3}a^3 f''(\nu)$$

for some (unknown) $-a \le \nu \le a$. [*Hint*: $f(x) = f(0) + f'(0)x + (1/2)f''(\xi)x^2$ where $\xi = \xi(x)$ is between 0 and x.] How accurate is the midpoint rule on the integral $\int_{-0.1}^{0.1} e^x\,dx$?

5. Using Euler's method for approximately solving ODEs, write your own Matlab program (either script or function) to solve initial value problem (1) to approximate $y(10)$. Adjust the step size so that the result is accurate to four digits. [*Hint*: $y'''+y''+y'+y = 0$ can, and should, be written as a system of three first order equations.]

6. Use Matlab's built in `ode45` to solve initial value problem (1) to approximate $y(10)$. [Same *hint* as in problem **5**!]