Quiz # 7 Solutions.

1.

$$\lim_{(x,y)\to(2,2)} \frac{x+y-4}{\sqrt{x+y}-2} = \lim_{(x,y)\to(2,2)} \frac{(\sqrt{x+y}-2)(\sqrt{x+y}+2)}{\sqrt{x+y}-2} = \lim_{(x,y)\to(2,2)} \sqrt{x+y}+2 = \sqrt{2+2}+2 = 4.$$

2.

$$\frac{\partial g}{\partial x} = e^x + \ln y + \frac{y}{x};$$
$$\frac{\partial g}{\partial y} = \frac{x}{y} + \ln x.$$

3. [*First I drew the x, y plane and noted that the domain is the whole plane except the origin. Then I saw that if $x = 0$ I get one constant, and if $y = 0$ I get another*:]

Along the path $x = 0$,

$$\lim_{y\to 0} \frac{0}{0+y^2} = 0.$$

Along the path $y = 0$,

$$\lim_{x\to 0} \frac{x^4}{x^4+0} = 1.$$

Thus the limit $(x, y) \to (0, 0)$ does not exist.

4. The level curves are all parabolas. For instance, if we consider $-2, -1, 0, 1, 2$ as output values, the level curves are

$$y = x^2 - 6; \quad y = x^2 - 5; \quad y = x^2 - 4; \quad y = x^2 - 3; \quad y = x^2 - 2.$$

Graphing these is easy.

In words, I would say the surface is a parabolic, slanted valley where the valley floor is the y–axis and one slides down the valley floor in the negative y direction. [*Many possible descriptions will get full credit.*]