

Quiz # 6 Solutions.

1. I get

$$\mathbf{v}(t) = -4 \sin t \mathbf{i} + 4 \cos t \mathbf{j} + 3 \mathbf{k},$$

$$|\mathbf{v}(t)| = \sqrt{4^2 + 3^2} = 5.$$

Thus

$$s = \int_0^t 5 \, d\tau = 5t.$$

2.

$$\text{domain} = \{(x, y) \mid x \neq 0\};$$

$$\text{range} = (0, \infty).$$

3. I sketched the level surface $1 = f(x, y, z)$, that is $1 = y^2 + z^2$. This is a cylinder which has radius 1 and has the x -axis along its axis.

4. I calculated:

$$\mathbf{T} = -(4/5) \sin t \mathbf{i} + (4/5) \cos t \mathbf{j},$$

$$\frac{d\mathbf{T}}{dt} = -(4/5) \cos t \mathbf{i} - (4/5) \sin t \mathbf{j},$$

$$\left| \frac{d\mathbf{T}}{dt} \right| = 4/5,$$

and

$$\kappa = \left| \frac{d\mathbf{T}}{ds} \right| = \frac{\left| \frac{d\mathbf{T}}{dt} \right|}{|\mathbf{v}|} = \frac{4/5}{5} = \frac{1}{25} = \frac{1}{6.25}.$$

The last expression is close to the meaning. That is, the osculating circle of the curve, at any point, has radius 6.25. Since, in fact, the curve is a helix which follows a cylinder of radius 4, this makes sense.