Quiz # 5 Solutions.

1. Here $\mathbf{v}(t) = \mathbf{i} + 2t\mathbf{j} + 3t^2\mathbf{k}$ so $\mathbf{v}(1) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{r}(1) = \mathbf{i} + \mathbf{j} + \mathbf{k}$. Thus the line is

$$\mathbf{l}(t) = \mathbf{r}(1) + \mathbf{v}(1)t = \langle 1 + \mathrm{t}, 1 + 2\mathrm{t}, 1 + 3\mathrm{t} \rangle.$$

2. Integrating gives

$$\mathbf{r}(t) = (t^3 + t^2)\mathbf{i} + \frac{1}{\pi}\sin(\pi t)\mathbf{j} + \left(\frac{t^2}{2} - t\right)\mathbf{k} + \mathbf{c}.$$

Using the fact $\mathbf{r}(1) = 3\mathbf{k}$ gives $\mathbf{c} = -2\mathbf{i} + \frac{7}{2}\mathbf{k}$ so

$$\mathbf{r}(t) = (t^3 + t^2 - 2)\mathbf{i} + \frac{1}{\pi}\sin(\pi t)\mathbf{j} + \left(\frac{t^2}{2} - t + \frac{7}{2}\right)\mathbf{k}.$$

3. Since $\mathbf{v}(t) = -2\sin t\mathbf{i} + 2\cos t\mathbf{j} + \mathbf{k}$,

$$L = \int_0^5 |\mathbf{v}(t)|\,dt = \int_0^5 \sqrt{(-2\sin t)^2 + (2\cos t)^2 + (1)^2}\,dt = \int_0^5 \sqrt{5}\,dt = 5\sqrt{5}.$$

4. [*As stated at the time of the quiz,* (c) *should be* "$0 = -z + x^2 + y^2$".]
(a) II
(b) IV
(c) I
(d) III

Also, I have had some questions about #14 in section 10.4, so here goes:

Recall that the problem is to find the volume of a barrel with dimensions r, R, h. I get

$$V = \frac{2}{3}\pi h\left(2R^2 + r^2\right).$$

In the cylinder case where $r = R$, this becomes

$$V \to V = 2\pi h R^2$$

which is correct since the total height is $2h$. In the sphere case where $r = 0$ and $h = R$,

$$V \to V = \frac{4}{3}\pi R^3,$$

which is correct.