Quiz # 4 Solutions.

1. *CORRECTED*: I find vectors corresponding to the three sides, then two of the angles and then the third angle from the fact that the total angle is π:

$$\mathbf{PQ} = \langle 2, 0, 2\rangle, \quad \mathbf{PR} = \langle 1, -2, 1\rangle, \quad \mathbf{QR} = \langle -1, -2, -1\rangle;$$
$$|\mathbf{PQ}| = 2\sqrt{2}, \quad |\mathbf{PR}| = \sqrt{6}, \quad |\mathbf{QR}| = \sqrt{6}.$$

Note $\mathbf{QP} = -\mathbf{PQ}$. So,

$$\theta_P = \cos^{-1}\left(\frac{\mathbf{PQ}\cdot\mathbf{PR}}{|\mathbf{PQ}||\mathbf{PR}|}\right) = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right),$$
$$\theta_Q = \cos^{-1}\left(\frac{-\mathbf{PQ}\cdot\mathbf{QR}}{|\mathbf{PQ}||\mathbf{QR}|}\right) = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right),$$
$$\theta_R = \pi - 2\cos^{-1}\left(\frac{1}{\sqrt{3}}\right).$$

2. The standard form is $\mathbf{n}\cdot\mathbf{PP_0} = 0$, so write

$$4(x-1) + (-1)(y-1) + (-2)(z-1) = 0$$

or

$$4x - y - 2z = 1.$$

3. The answer is to take a cross product and then normalize:

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ -2 & -2 & 0 \end{vmatrix} = 2\mathbf{i} - 2\mathbf{j} - 2\mathbf{k};$$
$$\hat{\mathbf{n}} = \frac{\mathbf{u}\times\mathbf{v}}{|\mathbf{u}\times\mathbf{v}|} = \frac{1}{\sqrt{3}}\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j} + \frac{1}{\sqrt{3}}\mathbf{k}.$$

4. The standard way to write $\mathbf{u}$ this way is:

$$\mathbf{u} = \mathrm{proj}_{\mathbf{v}}\,\mathbf{u} - (\mathbf{u} - \mathrm{proj}_{\mathbf{v}}\,\mathbf{u}).$$

Recall that $\mathrm{proj}_{\mathbf{v}}\,\mathbf{u} = \left(\mathbf{u}\cdot\frac{\mathbf{v}}{|\mathbf{v}|}\right)\cdot\frac{\mathbf{v}}{|\mathbf{v}|}$. Note $\frac{\mathbf{v}}{|\mathbf{v}|} = \left\langle -\frac{3}{5}, 0, \frac{4}{5}\right\rangle$. Thus

$$\mathrm{proj}_{\mathbf{v}}\,\mathbf{u} = \left(-\frac{2}{5}\right)\left\langle -\frac{3}{5}, 0, \frac{4}{5}\right\rangle = \left\langle \frac{6}{25}, 0, -\frac{8}{25}\right\rangle$$

and

$$\mathbf{u} - \mathrm{proj}_{\mathbf{v}}\,\mathbf{u} = \left\langle \frac{44}{25}, 1, \frac{33}{25}\right\rangle,$$

and

$$\mathbf{u} = \left\langle \frac{6}{25}, 0, -\frac{8}{25}\right\rangle + \left\langle \frac{44}{25}, 1, \frac{33}{25}\right\rangle.$$

[*You can check three facts very easily that tell you this is the right answer:* (i) *the given vectors sum to* $\mathbf{u} = \langle 2, 1, 1\rangle$, (ii) *the first is a multiple of* $\mathbf{v} = \langle -3, 0, 4\rangle$, (iii) *the last is orthogonal to* $\mathbf{v}$ *(use dot product to check).*]