Quiz # 2 Solutions.

1. From the equations of the lines, $\mathbf{v}_1 = \langle 2, 1\rangle$ and $\mathbf{v}_2 = \langle 1, -1\rangle$ are the normal vectors. Thus

$$\theta = \arccos\left(\frac{\mathbf{v}_1 \cdot \mathbf{v}_2}{|\mathbf{v}_1||\mathbf{v}_2|}\right) = \arccos\left(\frac{2-1}{\sqrt{5}\sqrt{2}}\right) = \arccos\left(\frac{1}{\sqrt{10}}\right),$$

which is close to $\frac{\pi}{2}$ but is acute.

2. [The graph is just $x = \cos y$. That is, graph $y = \cos x$ and then relabel the axes and look through the back of the paper to see the right graph.]

3. *One method* is to find the antiderivative and then use the given value to find $\mathbf{c}$:

$$\mathbf{r}(t) = \left(\frac{t^4}{4} - \frac{t^2}{2}\right)\mathbf{i} + \frac{t^2}{2}\mathbf{j} + \mathbf{c},$$

so $\mathbf{j} = \mathbf{r}(1) = (\frac{1}{4} - \frac{1}{2})\mathbf{i} + \frac{1}{2}\mathbf{j} + \mathbf{c}$, which implies $\mathbf{c} = \frac{1}{4}\mathbf{i} + \frac{1}{2}\mathbf{j}$, that is

$$\mathbf{r}(t) = \left(\frac{t^4}{4} - \frac{t^2}{2} + \frac{1}{4}\right)\mathbf{i} + \left(\frac{t^2}{2} + \frac{1}{2}\right)\mathbf{j}.$$

Another method is to do a definite integral:

$$\mathbf{r}(t) - \mathbf{r}(1) = \int_1^t (s^3 - s)\mathbf{i} + s\mathbf{j}\; ds = \left(\frac{t^4}{4} - \frac{t^2}{2}\right)\mathbf{i} + \frac{t^2}{2}\mathbf{j} + \frac{1}{4}\mathbf{i} - \frac{1}{2}\mathbf{j},$$

so using the given $\mathbf{r}(1)$ you get the same answer.

4. By taking derivatives, $\mathbf{v}(t) = \mathbf{i} - 3\sin t\mathbf{j}$ and $\mathbf{a}(t) = \mathbf{0} - 3\cos t\mathbf{j}$. Thus $\mathbf{v}\cdot\mathbf{a} = 0 + 9\sin t\cos t$. Solve

$$9\sin t\cos t = 0$$

for t to find when $\mathbf{v}$ is perpendicular (orthogonal) to $\mathbf{a}$.

That is, $t = 0, \pi$ makes $\sin t = 0$ and $t = \frac{\pi}{2}$ makes $\cos t = 0$, in the given range, so the answer is

$$t = 0, \frac{\pi}{2}, \pi.$$