Practice Quiz # 9.

1. Find the linearization $L(x, y)$ of the function at the point:

$$f(x, y) = e^{2y-x}; \qquad P_0 = (1, 2).$$

2. About how accurately may the volume of a cone be calculated from measurements r and h that are in error by 1%? [*Hint*: $V = \frac{1}{3}\pi r^2 h$.]

3. Find the absolute maxima and minima of the function on the given domain:

$f(x, y) = x^2 - 2x + y^2 - 2y$ on the triangular plate bounded by the lines $x = 0, y = 2, y = x$.

Answers to Practice Quiz # 9.

Solution 1. Here $f_x = -e^{2y-x}$ and $f_y = 2e^{2y-x}$ and $x_0 = 1, y_0 = 2$ so

$$\begin{aligned} L(x, y) &= f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) \\ &= e^3 + (-e^3)(x - 1) + (2e^3)(y - 2) = e^3(1 - (x - 1) + 2(y - 2)). \end{aligned}$$

Solution 2. The key is that dr and dh are known only *relatively*, that is, in percentage terms. Thus we calculate dV relatively, that is, dV/V:

$$dV = \frac{2}{3}\pi r h dr + \frac{1}{3}\pi r^2 dh,$$

$$\frac{dV}{V} = 2\frac{dr}{r} + \frac{dh}{h}.$$

In these terms we know $\left|\frac{dr}{r}\right| \le .01$, $\left|\frac{dh}{h}\right| \le .01$. Thus

$$\left|\frac{dV}{V}\right| \le 2(.01) + .01 = .03,$$

that is, V can be calculated within 3%.

Solution 3. First we find the critical points, if any, in the interior of the triangle:

$f_x = 2x - 2; \qquad f_y = 2y - 2; \qquad$ so the only critical point is $(1, 1)$, which is on the boundary.

We will deal with $(1, 1)$ as a boundary point.

Break the boundary into three pieces:

(1) $x = 0$, $0 \le y \le 2$;
(2) $y = 2$, $0 \le x \le 2$; and
(3) $x = y$, $0 \le x \le 2$.

Along (1) we find extremes of f, that is, we find the absolute max. and min. of $f_1(y) = y^2 - 2y$, $0 \le y \le 2$. Finding $f_1'(y)$ and setting it to zero, and comparing the values of f_1 at the critical and boundary points we get -1 and 0 as minimum and maximum.

Along (2) with $f_2(x) = x^2 - 2x$ we find the same minimum and maximum.

Along (3) with $f_3(x) = 2x^2 - 4x$ we find a minimum of -2 and maximum of zero. Note that the minimum is achieved when $x = 1$, thus at the critical point $(1, 1)$.

Thus the absolute maximum of f is 0, achieved at each corner of the triangle. The absolute minimum of f is -2, achieved at $(1, 1)$.