

Name: \_\_\_\_\_

Math 200 Calculus I (Bueler)

5 April 2006

## Midterm # 2

*Total of 100 points.*

1. (*8 pts*) A spherical balloon is being inflated. Find the rate of increase of the surface area with respect to the radius  $r$  when  $r = 3$ .

2. (*8 pts*) A spherical balloon is being inflated by blowing air into it at a rate of  $36 \text{ cm}^3/\text{sec}$ . Find the rate of change *in time* of the radius when the radius is 3 cm.

**3.** Find the derivatives.

**(a)** (6 pts)

$$y = \frac{x}{\cos x}$$

**(b)** (6 pts)

$$f(t) = \sqrt[3]{1 + \tan t}$$

**(c)** (6 pts)

$$g(x) = 3^{(x^2)}$$

4. (a) (7 pts) Find  $dy/dx$  by implicit differentiation:

$$x^3 + x^2y + 4y^2 = 6$$

(b) (7 pts) The point  $(1, 1)$  is on the curve in (a). Find the equation of the tangent line to this curve at this point.

5. (10 pts) Find the absolute maxima and minima of

$$f(x) = \frac{x}{x^2 + 1}$$

on  $[0, 2]$ .

6. (10 pts) Sketch a graph with the following properties:

$f$  is continuous on  $\mathbb{R}$ ,

$$f'(-1) = f'(1) = 0,$$

$$f''(x) < 0 \text{ if } x < 0, \quad f''(x) > 0 \text{ if } x > 0.$$

7. (a) (7 pts) Prove the formula

$$\frac{d}{dx} (\sec x) = \sec x \tan x$$

(b) (7 pts) Prove the formula

$$\frac{d}{dx} (\sec^{-1} x) = \frac{1}{x\sqrt{x^2 - 1}}$$

[Hint: Implicit differentiation.]

8. (a) (7 pts) Find the linearization  $L(x)$  of  $f(x) = \ln x$  at  $a = 1$ .

(b) (7 pts) Sketch the graphs  $y = f(x)$  and  $y = L(x)$  on the same axes.

(c) **Extra Credit** (3 pts) Use the linearization  $L(x)$  in part (a) to estimate  $\ln 4$ . Is the error smaller or larger than 1? than 2?

9. (4 pts) **Multiple choice.** The textbook claims in an exercise that if two runners start a race at the same time and finish in a tie then at some time during the race they have the same speed. Proving this is an application of one of the following theorems; **circle the right one.**

(i) Intermediate Value Theorem

(ii) Extreme Value Theorem

(iii) Mean Value Theorem