

Name: \_\_\_\_\_

Math 200 Calculus I (Bueler)

1 March 2006

## Midterm # 1

*Total of 100 points.*

1. (12 pts) Define, in *one* or *two* sentences, the statement

$$\lim_{x \rightarrow a} f(x) = L.$$

2. (a) (6 pts) Find formulas for the functions  $f \circ g$ ,  $f \circ f$ ,  $g \circ f$  if  $f(x) = \sin x$  and  $g(x) = (1 + x)^{-1}$ .

- (b) (6 pts) Determine the domains of  $f \circ g$ ,  $f \circ f$ ,  $g \circ f$  above.

**3.** Find the derivatives:

**(a)** (8 pts)

$$f(x) = 1 - x^2 + x^6 - x^{18}$$

**(b)** (8 pts)

$$g(x) = \frac{x^2}{x-1}$$

**4.** (10 pts) Find the limit:

$$\lim_{x \rightarrow \infty} \sqrt{x^2 + 1} - x$$

5. (10 pts) Use the given graph of  $f$  to sketch the graphs of  $f^{-1}$  (the inverse function of  $f$ ) and  $1/f$  on the same axes. Clearly label the two new graphs.

6. (10 pts) Find the limit:

$$\lim_{h \rightarrow 0} \frac{(1+h)^4 - 1}{h}$$

7. (10 pts) The graph of  $f$  is given at left below. Sketch the graph of  $f'$ .

8. (10 pts) Sketch the graph of an example function  $f$  that satisfies all the given conditions:

$$\begin{aligned} \lim_{x \rightarrow 2} f(x) = -\infty, & \quad \lim_{x \rightarrow \infty} f(x) = \infty, & \quad \lim_{x \rightarrow -\infty} f(x) = 0, \\ \lim_{x \rightarrow 0^+} f(x) = \infty, & \quad \lim_{x \rightarrow 0^-} f(x) = -\infty \end{aligned}$$

9. (10 pts) Find the equation of the tangent line to the curve at the given point:

$$y = 3x + \frac{1}{x}, \quad (1, 4)$$

**Extra Credit.** (4 pts) Show by a calculation that  $y = x^{2/3}$  has a vertical tangent line at  $(0, 0)$ .