Quiz # 9: SOLUTIONS

1. (a) The areas are $A_{\text{square}} = (\frac{x}{4})(\frac{x}{4})$ and $A_{\text{circle}} = \pi r^2$. But we know that the circumference of the circle is $C = 9 - x = 2\pi r$ so $r = \frac{9-x}{2\pi}$. Therefore the total area is

$$A(x) = \frac{x^2}{16} + \pi\left(\frac{9-x}{2\pi}\right)^2 = \frac{x^2}{16} + \frac{(9-x)^2}{4\pi}$$

(b) Take the derivative of A and then set $A' = 0$ and solve for x:

$$\begin{aligned} A' &= \frac{x}{8} + \frac{(9-x)(-1)}{2\pi} \\ 0 &= \frac{x}{8} + \frac{(9-x)(-1)}{2\pi} \\ \frac{9}{2\pi} &= x\left(\frac{1}{8} + \frac{1}{2\pi}\right) \\ x &= \frac{36}{\pi + 4} \end{aligned}$$

Note that $x = 36/(\pi+4) \approx 36/7 \approx 5$ is a critical number on the interval $0 \le x \le 9$. It must be the minimum because $A(x)$ is a quadratic function with positive coefficient on the x^2 term (so its graph is a parabola with a single minimum.

2. We know that $\frac{d}{dx}(\tan x) = \sec^2 x$. So $F(x) = 4e^x + 7\tan x + C$.

3.

$$\begin{aligned} f'(x) &= 9x^{\frac{1}{2}} + 4x^2 \\ f(x) &= \frac{18}{3}x^{\frac{3}{2}} + \frac{4}{3}x^3 + C = 6x^{\frac{3}{2}} + \frac{4}{3}x^3 + C \end{aligned}$$

But we know that $f(1) = 0$, so it follows that

$$0 = 6(1)^{\frac{3}{2}} + \frac{4}{3}(1)^3 + C$$

and therfore that $C = -\frac{22}{3}$. Hence,

$$f(x) = 6x^{\frac{3}{2}} + \frac{4}{3}x^3 - \frac{22}{3}.$$