Quiz # 8: SOLUTIONS

1. (a) For the vertical asymptotes just set the denominator equal to zero:

$$x^2 - 4 = 0 \to x^2 = 4 \to x = \pm 2$$

For the horizontal asymptotes we take the limit of the function as x approaches $\pm\infty$. So

$$\lim_{x\to\infty} \frac{e^{x/2}}{x^2-4} = \infty \qquad \text{and} \qquad \lim_{x\to-\infty} \frac{e^{x/2}}{x^2-4} = 0.$$

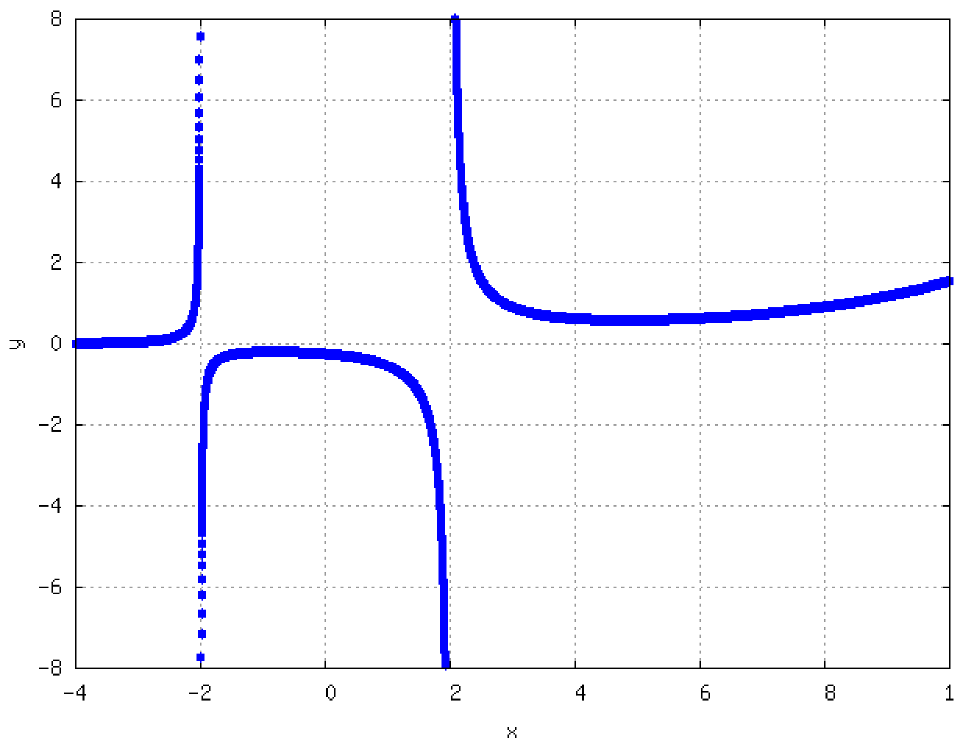
There are vertical asymptotes at $x = 2$ and $x = -2$, and a horizontal asymptote at $y = 0$ (as $x \to -\infty$.)

(b) Start by taking the derivative of the function by using the quotient rule

$$y' = \frac{\frac{1}{2}e^{x/2}(x^2-4) - 2xe^{x/2}}{(x^2-4)^2} = \frac{e^{x/2}(x^2-4x-4)}{2(x^2-4)^2}.$$

Setting $y' = 0$ is the same as $e^{x/2}(x^2-4x-4) = 0$. Since $e^{x/2} \neq 0$ then we just have $0 = x^2 - 4x - 4$ and using the quadratic equation we get $x = 2 \pm 2\sqrt{2}$. Hence the critical numbers are $x = 2 \pm 2\sqrt{2}$.

(c)



2. The limit is not easy because

$$\lim_{x\to 0} \frac{\sin^2 x}{1-\cos x} = \text{“}\ \frac{0}{0}\ \text{”}.$$

We may use L'Hospital rule:

$$\lim_{x\to 0} \frac{2\sin x\cos x}{0-(-\sin x)} = \lim_{x\to 0} 2\cos x = 2$$