Quiz # 7: SOLUTIONS

1. We start by taking the derivative of $g(y) = \dfrac{2y-1}{y^2+2}$ using the quotient rule. So,

$$g'(y) = \frac{2(y^2+2) - 2y(2y-1)}{(y^2+2)^2} = \frac{2y^2+4-4y^2+2y)}{(y^2+2)^2} = \frac{-2y^2+2y+4}{(y^2+2)^2}$$

Find the critical numbers by setting the fraction, i.e. its top, to zero:

$$\begin{aligned} 0 &= -2y^2 + 2y + 4 \\ &= y^2 - y - 2 \\ &= (y-2)(y+1). \end{aligned}$$

Hence the critical numbers are $y = -1, 2$.

2. To start with we know that $y(0) = 3000$ bacteria and after 8 hours we have a population size of $y(8) = 60000$ bacteria. Using the equation $y(t) = y(0)e^{kt}$ we have

$$60000 = 3000e^{8k}.$$

This simplifies to

$$k = \frac{\ln(20)}{8}.$$

3. The volume of the snowball is $V = \frac{4}{3}\pi r^3$ where r is the radius. We are told that

$$\frac{dr}{dt} = -\frac{1}{2}\ \frac{\text{cm}}{\text{min}}.$$

Now we need to take the derivative of the volume with respect to time:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Specifically we want the rate of change of the volume at the time when $r = 5$ cm:

$$\frac{dV}{dt} = 4\pi 5^2 \left(-\frac{1}{2}\right) = -50\,\frac{\text{cm}^3}{\text{min}}.$$