MATH 200 (Bueler/Buchholz) March 28, 2008

Quiz # 6: SOLUTIONS

1. *Find dy/dx by implicit differentiation $x^2y + \cos y = 4$.*

Solution.

$$\begin{aligned} \frac{d}{dx}(x^2y + \cos y) &= \frac{d}{dx}(4) \\ 2xy + x^2\frac{dy}{dx} + (-\sin y)\frac{dy}{dx} &= 0 \\ \frac{dy}{dx}(x^2 - \sin y) &= -2xy \\ \frac{dy}{dx} &= \frac{2xy}{\sin y - x^2} \end{aligned}$$

2. *Show using implicit differentiation that*

$$\frac{d}{dx}[\arcsin(x)] = \frac{1}{\sqrt{1-x^2}}.$$

Start by writing $y = \arcsin x$ as $\sin y = x$.

Solution. Differentiate both sides of the equation $\sin y = x$ with respect to x and get

$$\cos y\frac{dy}{dx} = 1 \qquad \text{or} \qquad \frac{dy}{dx} = \frac{1}{\cos y}.$$

But

$$\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1-x^2}.$$

Thus,

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}.$$

(Note that the range of $\arcsin x$ is $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. Therefore $\cos y \ge 0$. So we choose the positive square root: $\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-x^2}$.)

3. *Differentiate $f(x) = \ln\left(\frac{\sqrt{x}}{2x+1}\right)$.*

Solution. There are two ways in which to solve this problem, as follows.

- You can differentiate before simplifying:

$$f'(x) = \left(\frac{1}{\frac{\sqrt{x}}{2x+1}}\right)\left(\frac{(\frac{1}{2}x^{-\frac{1}{2}})(2x+1) - (\sqrt{x})(2)}{(2x+1)^2}\right) = \left(\frac{1}{\sqrt{x}}\right)\left(\frac{1}{2\sqrt{x}} - \frac{2\sqrt{x}}{2x+1}\right) = \frac{1}{2x} - \frac{2}{2x+1}$$

- You can simplify by applying logarithm rules and then differentiate:

$$f(x) = \ln\left(\frac{\sqrt{x}}{2x+1}\right) = \ln(\sqrt{x}) - \ln(2x+1) = \frac{1}{2}\ln(x) - \ln(2x+1)$$

  so

$$f'(x) = \frac{1}{2x} - \frac{2}{2x+1}.$$

4. *Find the instantaneous rate of change of the area A of a circle with respect to its radius r when $r = 7$.*

Solution.

$$A = \pi r^2 \qquad \text{so} \qquad \frac{dA}{dr} = 2\pi r = 2\pi 7 = 14\pi.$$