MATH 200 (Bueler/Buchholz) March 25, 2008

Quiz # 5: SOLUTIONS

1. *The question was ambiguous*: "Differentiate $y = \csc\theta(\theta + \cot\theta)$." *Sorry!*

Some students (validly!) interpreted y as

$$y = \csc\Big(\theta(\theta + \cot\theta)\Big)$$

In this case the derivative requires applying the chain rule first. The solution:

$$y' = [-\csc(\theta(\theta + \cot\theta))\cot(\theta(\theta + \cot\theta))][2\theta + \cot\theta - \theta\csc^2\theta]$$

Other students (also validly!) interpreted y as

$$y = \Big(\csc\theta\Big)(\theta + \cot\theta)$$

In this case the product rule applies first:

$$y' = \csc\theta - \theta\csc\theta\cot\theta - \csc^3\theta - \csc\theta\cot^2\theta$$

If either answer was given correctly then it got full credit.

2. Just use the chain rule:

$$f'(x) = e^{\sin x}\cdot\cos(x)$$

3. Make $f(t) = \sqrt{g(t)}$ where $g(t) = t + \sqrt{h(t)}$ and $h(t) = t + \sqrt{t}$. Then

$$f'(t) = \frac{1}{2}(g(t))^{-\frac{1}{2}}\left[1 + \frac{1}{2}(h(t))^{-\frac{1}{2}}\left(1 + \frac{1}{2}t^{-\frac{1}{2}}\right)\right]$$

Then substituting back in $g(t)$ and $h(t)$ we get,

$$f'(t) = \frac{1}{2}\left(t + \sqrt{t + \sqrt{t}}\right)^{-\frac{1}{2}}\left[1 + \frac{1}{2}\left(t + \sqrt{t}\right)^{-\frac{1}{2}}\left(1 + \frac{1}{2}t^{-\frac{1}{2}}\right)\right]$$

4. Write y as $y = 2(1 + e^{-x})^{-1}$. Differentiate:

$$y' = -2(1 + e^{-x})^{-2}(-e^{-x}).$$

Then, putting in $x = 0$, we have $m = y'(0) = \frac{1}{2}$. Note $y(0) = 2/2 = 1$. The tangent line is $y - y_0 = m(x - x_0)$ or

$$(y - 1) = \frac{1}{2}(x - 0)$$

or

$$y = \frac{1}{2}x + 1.$$