Quiz # 4: SOLUTIONS

1. Rewrite the function as $f(x) = Ax^{-10} + Be^x$. Then differentiate using the power rule and $\frac{d}{dx}e^x = e^x$:

$$f'(x) = -10Ax^{-11} + Be^x.$$

2. To differentiate $f(x) = x^{\frac{5}{2}}e^x$ you will need to use the product rule:

$$f'(x) = \frac{5}{2}x^{\frac{3}{2}}e^x + x^{\frac{5}{2}}e^x$$
$$f''(x) = \frac{15}{4}x^{\frac{1}{2}}e^x + \frac{5}{2}x^{\frac{3}{2}}e^x + \frac{5}{2}x^{\frac{3}{2}}e^x + x^{\frac{5}{2}}e^x = \frac{15}{4}x^{\frac{1}{2}}e^x + 5x^{\frac{3}{2}}e^x + x^{\frac{5}{2}}e^x$$

3. Find the derivative of $y = x^3 - 3x + 1$, which is $y' = 3x^2 - 3$. Then by setting the derivative equal to zero you will find where the tangent line is horizontal:

$$0 = 3x^2 - 3$$
$$0 = 3(x^2 - 1)$$
$$x = \pm\sqrt{1} = \pm 1.$$

The points in the (x, y) plane at which the tangent line is horizontal are (1,-1) and (-1,3).

4. First note that $\tan x = \frac{\sin x}{\cos x}$. To find the derivative we must use the quotient rule and the trig identity $\cos^2 x + \sin^2 x = 1$:

$$\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{(\cos x)^2} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x.$$

5. To find the equation of the tangent line to the graph $y = \sin x$ at the point with x-coordinate $x = \frac{\pi}{4}$, first we find the y-coordinate of the point, $y_0 = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$. Next we take the derivative of the function to find the slope of the tangent line:

$$y' = \cos x \qquad \Rightarrow \qquad m = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}.$$

Lastly use the point-slope formula for a line:

$$\left(y - \frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right).$$