Quiz # 10: SOLUTIONS

1. First graph $y = 2x - 3$. The integral involves two triangles, one below the x-axis and one above the x-axis. The one below has an area of $\frac{1}{2}(2.5)(5) = \frac{25}{4}$ while the one above the axis has an area of $\frac{1}{2}(.5)(1) = \frac{1}{4}$, so

$$\int_{-1}^{2} (2x - 3)dx = -\frac{25}{4} + \frac{1}{4} = -6.$$

2. Note $\Delta x = \frac{4-0}{4} = 1$ and $x_i = 0 + i\Delta x = i$. So the Riemann sum is

$$\sum_{i=1}^{4} f(x_i)\Delta x = 1 \cdot (f(1) + f(2) + f(3) + f(4)) = 2 + 5 + 12 + 17 = 34.$$

3. Let $\Delta x = \frac{2-0}{n} = \frac{2}{n}$. The integral is, by definition, this limit of Riemann sums:

$$\int_0^2 \frac{x}{2 + x^4}\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{x_i^*}{2 + (x_i^*)^4}\, \frac{2}{n}$$

where x_i^* is in the ith subinterval. Choice of left or right endpoints is fine; the right endpoints, for example, give

$$\int_0^2 \frac{x}{2 + x^4}\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2i/n}{2 + (2i/n)^4}\, \frac{2}{n}$$

4. Note that the integral is to be evaluated from 2 to 2. Hence

$$\int_2^2 \frac{\arcsin(e^x)}{2 + \cos(x^4)} dx = 0.$$