

NAME: _____

MATH 200 Calculus 1 (Bueler)

9 April, 2008

Midterm Exam # 2

100 points total. You have 60 minutes.

1. Find the derivatives.

(a) (6 pts)

$$f(t) = \sqrt{t} - \frac{1}{\sqrt{t}}$$

(b) (6 pts)

$$f(x) = (x^3 + 2x)e^{x^2}$$

(c) (6 pts)

$$y = \log_{10}(x^2 + 1)$$

(d) (6 pts)

$$y = \frac{\sin x}{e^x + \cot x}$$

(e) (6 pts)

$$h(t) = \ln \left(\frac{t\sqrt{t^2 + 1}}{t - 1} \right)$$

2. In each case, find an equation of the tangent line to the curve at the given point.

(a) (9 pts)

$$y = \sec x, \quad (\pi, -1)$$

(b) (9 pts)

$$x^2 + xy + y^4 = 7, \quad (2, 1)$$

3. (10 pts) Use implicit differentiation to derive this formula:

$$\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$$

(Note $\arccos x = \cos^{-1} x$.)

4. The position of a particle is given by $s(t) = t^3 - 4.5t^2 - 7t$ for $t \geq 0$, where t is measured in seconds and s in meters.

(a) (6 pts) When does the particle reach a velocity of 5 m/s?

(b) (6 pts) When does the particle switch from decelerating to accelerating? (Note that “deceleration” means that the acceleration is negative.)

5. (10 pts) Sketch a graph of a function that satisfies these conditions:

$$\begin{aligned} f'(x) &> 0 \text{ if } |x| < 2, & f'(x) < 0 \text{ if } |x| > 2, \\ f'(-2) &= 0, & \text{vertical asymptote at } x = 2, & f''(x) > 0 \text{ if } x \neq 2. \end{aligned}$$

6. (10 pts) Find the absolute minimum and maximum of

$$f(x) = \frac{x}{x^2 + 1}$$

on the interval $[0, 5]$.

7. (10 pts) The top of a twelve foot long ladder is sliding down the wall while its base is sliding on the floor steadily away from the wall at one foot per second. At what speed is the top of the ladder sliding when the base of the ladder is six feet from the wall?

Extra Credit. (3 pts) Sketch the graph of a function which is defined on the whole real line, has an absolute maximum value of 1 and an absolute minimum value of -1 , for which every integer $x = n$ is a critical number, and which *never* has slope zero.