

Midterm Exam # 1: SOLUTIONS

1. (a) $(f \circ g)(x) = \sqrt{x^3 + 2}$

(b) x is in the domain of $f \circ g$ as long as we are not taking the square root of a negative number. That is, the domain is when $x^3 + 2 \geq 0$ or when $x \geq (-2)^{1/3}$. As an interval, the domain is $[(-2)^{1/3}, +\infty)$.

2.

$$\lim_{x \rightarrow +\infty} \frac{x^2 + x - 6}{2 - 4x^2} = \lim_{x \rightarrow +\infty} \frac{(x^2 + x - 6) \frac{1}{x^2}}{(2 - 4x^2) \frac{1}{x^2}} = \lim_{x \rightarrow +\infty} \frac{1 + \frac{1}{x} - \frac{6}{x^2}}{\frac{2}{x^2} - 4} = \frac{1 + 0 - 0}{0 - 4} = -\frac{1}{4}.$$

3. *SOLUTION 1.* The statement means that the values of $f(x)$ can be made as close as desired to L by choosing x sufficiently close to, but not equal to, a .

SOLUTION 2. The statement means that for every $\epsilon > 0$ there is a $\delta > 0$ so that

$$\text{if } 0 < |x - a| < \delta \quad \text{then} \quad |f(x) - L| < \epsilon.$$

4. (b) is correct

5.

$$\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x \rightarrow 2} \frac{(x + 3)(x - 2)}{x - 2} = \lim_{x \rightarrow 2} x + 3 = 2 + 3 = 5.$$

6. (a) *Sketch not given here.*

(b) The points of discontinuity are at $x = 0$ and at $x = 2$.

7. *Sketch not given here. There are many correct answers.*

8.

$$\begin{aligned} f'(3) &= \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{[3(3+h) - 1] - [3(3) - 1]}{h} \\ &= \lim_{h \rightarrow 0} \frac{9 + 3h - 1 - 9 + 1}{h} = \lim_{h \rightarrow 0} \frac{3h}{h} = 3 \end{aligned}$$

Note the consistent use of the equality sign. Note also the appearance of “ $\lim_{h \rightarrow 0}$ ” in every place there is a limit.

9. (a)

$$v_{av} = \frac{6.8 - 3.2}{3.0 - 1.2} = \frac{3.6}{1.8} = 2$$

(b) The instantaneous velocity *is* the derivative. If there is a formula for the hiker's speed, $s = p(t)$, then $p'(t)$ is the instantaneous velocity. Thus the instantaneous velocity at $t = 3.0$ is $p'(3) = 1.7$, with units of miles per hour in this case.

10. *Sketch not given here. There are many correct answers.*