Name: SAMPLE

Math 200 Calculus I (Bueler) 9 May 2006

Final Exam: Recent Part

Total of 100 points.

1. (*8 pts*) Compute the limit

$$\lim_{t\to 0} \frac{e^{3t}-1}{t} =$$

2. (*10 pts*) At which points on the curve $y = 8x^3 - \frac{3}{5}x^5$ does the tangent line have the largest slope? [Hint: *Don't* attempt to graph the curve.]

3. **(a)** (*8 pts*) Graph on the axes at right, clearly indicating any asymptotes in particular:

$f(x) = 5 + \dfrac{1}{\sqrt[3]{x}}$

(b) (*5 pts*) Find an antiderivative of $f(x)$ in part **(a)**.

(c) (*8 pts*) Graph the antiderivative found in part **(b)** on the axes at right.

4. **(a)** (*10 pts*) Find the definite integral *using the definition* and right endpoints:

$$\int_{-1}^{2} 1 - x^2 \, dx =$$

[Recall that $\sum_{i=1}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1)$.]

(b) (*5 pts*) Compute the same integral as in **(a)** using the Fundamental Theorem of Calculus.

5. Compute the integrals:

(a) *(10 pts)*

$$\int xe^{-x^2}\,dx =$$

(b) *(10 pts)*

$$\int_1^2 \frac{t}{1+t}\,dt =$$

6. **(a)** (*8 pts*) Find the area under the curve $y = 3 + \frac{1}{2}\sin x$ on $0 \leq x \leq \pi$.

(b) (*8 pts*) Graph the curve in **(a)**. Draw two rectangles whose areas are greater and less, respectively, than your area in **(a)**. Give their areas and thereby show that your answer in **(a)** is reasonable.

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Math 200 Calculus I (Bueler) 9 May 2006

Final Exam: Comprehensive Part

Total of 100 points.

1. (a) (*7 pts*) Find the derivative of $f(x) = e^x \sin(e^x)$.

(b) (*8 pts*) Find the antiderivative of $f(x) = e^x \sin(e^x)$.

2. Given that

$$\lim_{x\to a} f(x) = 2, \qquad \lim_{x\to a^-} g(x) = 0, \qquad \lim_{x\to a^+} g(x) = 5, \qquad \lim_{x\to a} h(x) = 0,$$

evaluate:

(a) *(5 pts)*

$$\lim_{x\to a^-} \frac{g(x)}{f(x)} =$$

(b) *(5 pts)*

$$\lim_{x\to a} \frac{h(x)+f(x)}{g(x)} =$$

(c) *(5 pts)*

$$\lim_{x\to a} h(x)f(x) =$$

3. *(10 pts)* Find the equation of the tangent line to $y = \tan x$ at $x = -\pi/4$.

4. (*10 pts*) Use the definition of the derivative to find $f'(3)$ if $f(x) = x + x^{-1}$.

5. (*10 pts*) Two cars start at the same point and one goes straight south at 30 mi/hr while the other goes straight east at 40 mi/hr. At what rate is the distance between the cars increasing at one hour?

6. (*10 pts*) Find the absolute minima and maxima of

$$f(x) = \frac{x}{x^2+1}$$

on the interval $[-5, 5]$.

7. (*10 pts*) Compute the definite integral

$$\int_1^2 \frac{x}{x^2+1}\, dx =$$