Quiz # 9 Solutions

1. Let $f(x) = x^4 - 1 - x$. We are trying to solve $f(x) = 0$, so, using $x_1 = 1$, Newton's method applies:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{1^4 - 1 - 1}{4 \cdot 1^3 - 1} = 1 - \frac{-1}{3} = 4/3 = 1.3333\ldots$$

Note $f'(x) = 4x^3 - 1$. [*It turns out the root is* $x = 1.2207440846058$*, to 14 digits, and Newton's method gets there at* x_6*.*]

2.

$$\sum_{i=1}^{79} = 1 + 2 + 3 + \cdots + 78 + 79 = \frac{79(79+1)}{2} = \frac{79 \cdot 80}{2} = 79 \cdot 40 = 3160,$$

that is, use $n = 79$ in the formula you know.

3.

$$G(x) = 3e^x + 7\tan x + C$$

is the (most general) antiderivative of $g(x) = 3e^x + 7\sec^2 x$.

4. **(a)** I am asking for what the text calls "R_3." Note $\Delta x = (2-(-1))/3 = 1$ and $x_i = a + i\Delta x = -1 + i$. Thus:

$$R_3 = \sum_{i=1}^{3} f(x_i)\Delta x = f(0)\cdot 1 + f(1)\cdot 1 + f(2)\cdot 1 = (1+0) + (1+1) + (1+4) = 8$$

because $f(x) = 1 + x^2$.

(b) The estimate would be *larger*. The point is that on the interval I specified $f(x)$ is increasing, and I said *right* endpoints. That means that the area of each rectangle would be found be using the largest possible f value for its height, and, in fact, each rectangle would over-estimate the desired area.