Quiz # 8 Solutions

1.

$$\lim_{t\to 0} \frac{e^{3t}-1}{t} \overset{\text{L'Hopital's}}{=} \lim_{t\to 0} \frac{3e^{3t}}{1} = 3.$$

[*The L'Hopital's rule step is justified because* $e^{3\cdot 0}-1 = 1-1 = 0$*; also the denominator is zero when* $t=0$.]

2. (a) The domain is the set of all $x \neq \pm 3$. The function is *even*.

(b) Since $y = (x^2-9)^{-1}$,

$$\frac{dy}{dx} = -(x^2-9)^{-2}(2x) = \frac{2x}{x^2-9} \qquad \text{so} \qquad \frac{2x}{x^2-9} = 0 \iff x = 0.$$

The only critical point is $x = 0$.

(c) There are vertical asymptotes $x=-3$ and $x=3$. Note that

$$\lim_{x\to\infty} \frac{1}{x^2-9} = 0$$

so $y=0$ is a horizontal asymptote.

(d) [*A sketch will be given in class.*]

3. The box has square base with sides l and height h. Thus the volume is $V = l^2h$, so

$$32 = l^2h.$$

On the other hand we want to minimize surface area, namely the material used in the box. There are five rectangular sides and we add up their areas:

$$S = l^2 + 4\cdot lh.$$

Let's eliminate h using the first equation:

$$h = \frac{32}{l^2}$$

so

$$S(l) = l^2 + 4l\left(\frac{32}{l^2}\right) = l^2 + 128\, l^{-1}.$$

Note that as $l \to 0^+$ and as $l \to +\infty$ the area $S(l)$ gets as big as you want. Thus the minimum will occur at a critical point, and not at an endpoint of the interval $(0,+\infty)$.

Find the critical point(s):

$$S'(l) = 2l - 128l^{-2} = 0 \qquad \iff \qquad l^3 = 64 \qquad \iff \qquad l = 4.$$

This is the only critical point. Thus $l = 4$ ft for the minimum area box; also $h = 32/l^2 = 2$ ft. (*And* $S(4) = 48\,\text{ft}^2$*, but you were asked for the dimensions not the resulting area.*)