Quiz # 6 Solutions

1. *Find dy/dx by implicit differentiation: $x^2 \ln x + x \sin y = 4$.*

Solution:

$$2x \cdot \ln y + x^2 \cdot \frac{1}{y}\frac{dy}{dx} + 1 \cdot \sin y + x \cdot \cos y \, \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-\left(2x \ln y + \sin y\right)}{x^2 y^{-1} + x \cos y}.$$

[*There is no need to simplify. There are many correct forms of the answer, of course.*]

2. **(a)** *Find the velocity and acceleration if $s(t) = 2t^3 - 15t^2 + 36t + 2$, $t \geq 0$.*

Solution:

$$v(t) = \frac{ds}{dt} = 6t^2 - 30t + 36,$$

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 12t - 30.$$

(b) *For the equation of motion in the first part, find the acceleration at the instants when the velocity is zero.*

Solution: $v(t) = 0$ is the same as $6t^2 - 30t + 36 = 0$ or

$$t^2 - 5t + 6 = 0$$

$$(t-2)(t-3) = 0$$

$$t = 2, \quad t = 3$$

The desired accelerations are

$$a(t{=}2) = a(2) = 12 \cdot 2 - 30 = -6,$$

$$a(t{=}3) = a(3) = 12 \cdot 3 - 30 = +6.$$

3. *Use implicit differentiation to find the equation of the tangent line at the given point: $x^2 + xy + y^2 = 3$, $(1,1)$ (an ellipse).*

Solution:

$$2x + 1 \cdot y + x \cdot \frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$$

so the line has slope $m = \frac{dy}{dx}\big|_{(1,1)} = -\frac{3}{3} = -1$ and the equation is

$$y - 1 = (-1)(x - 1) \qquad \text{or} \qquad y = -x + 2.$$

4. *Find the derivative: $f(t) = \tan^{-1}(2t + 1)$.*

Solution:

$$f'(t) = \frac{1}{(2t+1)^2 + 1} \cdot 2 = \frac{2}{4t^2 + 4t + 2} = \frac{1}{2t^2 + 2t + 1}.$$