

Quiz # 5 Solutions

1. Prove that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

Solution:

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{0 \cdot \cos x - 1 \cdot (-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos x \cdot \cos x} = \sec x \tan x.$$

2. (a) Find the derivative: $f(x) = \sec x / (x - \cot x)$.

Solution:

$$f'(x) = \frac{(\sec x \tan x)(x - \cot x) - (\sec x)(1 + \csc^2 x)}{(x - \cot x)^2}.$$

[There is no need to simplify, though you may.]

(b) Find the derivative: $y = \cos(e^x)$.

Solution:

$$\frac{dy}{dx} = -\sin(e^x) \cdot e^x.$$

3. The position function of a particle is given by $s = t^3 - 4.5t^2 - 7t$, where s is measured in meters and t is the (positive) time measured in seconds. When does the particle reach a velocity of 5 m/s?

Solution:

$$\dot{s} = 3t^2 - 9t - 7.$$

The velocity is \dot{s} . When is $\dot{s} = 5$?:

$$3t^2 - 9t - 7 = 5$$

$$3t^2 - 9t - 12 = 0$$

$$t^2 - 3t - 4 = 0$$

$$(t + 1)(t - 4) = 0$$

So $t = -1$ or $t = 4$. But we want the *positive* time only. Thus the answer is $t = 4$ seconds.

4. Find the points on the graph of $y = x + 2 \cos x$ where the tangent line is horizontal.

Solution:

$$\frac{dy}{dx} = 1 + 2(-\sin x) = 0$$

$$\sin x = \frac{1}{2}$$

so

$$x = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad x = \frac{5\pi}{6} + 2k\pi$$

for some integer k .